X

# From formal logic to formal ontology: The new dual paradigm in natural sciences

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In a visionary paper published almost forty years ago J.A. Wheeler posed the provocative question: "is physics legislated by cosmogony?" in front of the "quantum information revolution", related to the theory of "quantum computing" in fundamental physics and cosmology, a theory originally developed by the Nobel Laureate R. Feynman – the most famous of Wheeler students. The positive answer to such a question implies a deep revision of the ontology underlying the Newtonian physics of which best formal version is certainly R. Carnap's Logical Atomism (LA). The present work has thus a double, related issue. On one side, we present a first formal treatment of the Natural Realism (NR), as the proper formal ontology of the actual evolutionary cosmology. I.e., an ontology of the causal foundation of the same mathematical laws of physics, given that they evolve with the universe they rule. An issue for which some theoretical physicists and mathematicians tried to develop, at the foundation level, the theory of "arboreal causal sets". NR is thus systematically, formally different, despite several phenomenological contact points, from the analogue proposal of a naturalistic alternative to LA: the Conceptual Natural Realism (CNR), recently proposed by my colleague and friend, N. B. Cocchiarella. NR ontology, is based, indeed, on the logic of the converse implication ($q \leftarrow p$) and of its

modal version ($\neg\Diamond(q\wedge\neg p)$), as the logic of the formal causality, according to an Aristotle and Aquinas suggestion. In it, the truth in the inferential chain is not conserved, and hence it is the proper logic of the unpredictable emergence of coherent behaviors in which the individuality of the elements composing the system at the beginning of the process disappears, so to justify the emergence of collective behaviors, and hence of ever more complex structures. We demonstrate thus that the proper Modal Logic (ML) of NR is **KD45**, or **secondary S5**, and its Quantified ML (QML) is a possibilist first-order version (because of the axiom **D**) of the "objectual" **Q1R system**. In such a way, it is possible to formalize in NR an "arboreal" unraveling procedure of causal constitution (ancestor-descendants) – effectively a non-actualist version of R. Hayaki's "stipulation principle" - of nested domains/sub-domains of possible worlds, implementing a principle of "iterated modality" and of "stratified rigidity". In it, each level of the "unraveling" of equivalent domains has a **KD45** structure, and the whole system has a nested **KD45** structure, of growing complexity. NR seems thus an optimal candidate as formal ontology of an evolutionary cosmology based on the Quantum Field Theory (QFT), as irreducible to Quantum Mechanics (QM) because in the former, differently from the latter, the Stone-Von Neumann theorem of the finitely many unitarily equivalent commutation relations between quantum variables does not hold.

## 1. A change of paradigm: from Quantum Mechanics (QM) to Quantum Field Theory (QFT)

### *1.1 "Is physics legislated by cosmogony?"*

Perhaps, the better synthesis of the actual change of paradigm in fundamental physics is the positive answer that it seems necessary to give to the following question: "Is physics legislated by cosmogony?" Such a question is the title of a visionary paper wrote in 1975 by J. A. Wheeler and C. M. Patton and published in the first volume of a fortunate series of the Oxford University about the quantum gravity (Patton & Wheeler, 1975).

Such a revolution originally amounts to the so-called *information theoretic approach* in quantum physics as the natural science counterpart of a dual ontology taking information and energy as two fundamental magnitudes in basic physics and cosmology. This approach started from Richard Feynman's influential speculation that all of physics could be simulated by a quantum computer (Feynman, 1982), and from the famous "it from bit" ontological principle stated by his teacher J. A. Wheeler universe (Wheeler, 1990, p. 75). The cornerstones of this reinterpretation are, however, D. Deutsch's demonstration of the universality of the Quantum Universal Turing Machine (QTM) (Deutsch, 1985), and overall C. Rovelli's development of a *relational* QM (Rovelli, 1996). An updated survey of such an informational approach to fundamental physics is in the recent collective book edited by H. Zenil and with contributions, among the others, of R. Penrose, C. Hewitt, G. J. Chaitin, F. A. Doria, E. Fredkin, M. Hutter, S.Wolfram, S. Lloyd, besides the same D. Deutsch (Zenil, 2013).

There are, however, several theoretical versions of the information theoretic approach to quantum physics. It is not important to discuss all of them here (for an updated list in QM, see, for instance (Fields, 2012)), even though all can be reduced to essentially two.

The first one is related to a classical "infinitistic" approach to the *mathematical physics* of information in QM. Typical of this approach is the notion of the *unitary evolution* of the *wave function,* with the connected, supposed *infinite* amount of information it "contains", "made available" in different spatio-temporal cells via the mechanism of the "decoherence" of the wave function. Finally, essential for this approach is the necessity of supposing *an external observer* ("information for whom?" (Fields, 2012)) for the foundation of the notion and of the measure of information, reduced to the only Shannon's, purely syntactic, measure and notion of information in QM (Rovelli, 1996). Among the most prominent representatives of such an approach, we can quote the German physicist H. D. Zeh (Zeh, 2004; 2010) and the Swedish physicist at the Boston MIT, M. Tegmark (Tegmark, 2011).

The second approach, the emergent one today, is related to a "finitistic" approach to the *physical mathematics* of information, taken

as a fundamental physical magnitude together with energy. It is related to QFT, because of the possibility it gives of spanning the microphysical, macrophysical, and even the cosmological realms, within one only quantum theoretical framework, differently from QM (Blasone, Jizba, & Vitiello, 2011) (See also: (Lloyd, 2006; 2010; Davies, 2010; Smoot, 2010).

### *1.2 From QM to QFT in fundamental physics*

The theoretical, core difference between the two approaches can be essentially reduced to the criticism of the classical interpretation of the QFT as a "second quantization" as to the QM. In QFT, indeed, the classical Stone-Von Neumann theorem (Von Neumann, 1955) does not hold. This theorem states that, for system with a *finite* number of degrees of freedom, which is always the case in QM, the representations of the canonical commutation relations are all *unitarily equivalent to each other*, so to justify the exclusive use of Shannon information in QM.

On the contrary, in QFT systems, the number of the degrees of freedom is not finite, "so that infinitely many unitarily inequivalent representations of the canonical commutation (bosons) and anti-commutation (fermions) relations exist". Indeed, through the principle of the *Spontaneous Symmetry Breaking* (SSB) in the ground state, infinitely (not denumerable) many, quantum vacua conditions, compatible with the ground state, there exist. Moreover, this holds not only in the relativistic (microscopic) domain, but also it applies to non-relativistic many-body systems in condensed matter physics, i.e., in the macroscopic domain, and even on the cosmological scale (Blasone, Jizba, & Vitiello, 2011, p. 18. 53-96).

Indeed, starting from the discovery, during the 60's of the last century, of the dynamically generated long-range correlations mediated by the Nambu-Goldstone bosons (Goldstone J. , 1961; Goldstone, Salam, & Weinberg, 1962), and hence of their role in the local gauge theory by the Higgs field, the discovery of these collective modes changed deeply the fundamental physics. Before all, it appears as an effective, alternative method to the classically Newtonian paradigm of the perturbation theory, and hence to its postulate of the

asymptotic condition. In this way, the spontaneous breakdown of quantum vacuum symmetry; the thermal field theory; the phase transitions in a variety of problems at any scale; the process of defect formation during the process of non-equilibrium symmetry breaking in the phase transitions, characterized by an order parameter; all these phenomena and many others, can be studied by using the same approach of the inequivalent representations in QFT.

The emerging picture for the naturalistic ontology is thus deeply different from the atomism of the Newtonian one, as much the notion of mechanical vacuum is different from the notion of quantum vacuum. The ontological paradigm of physical system is no longer the isolated particle in the mechanical vacuum (= atomism) of which Carnap's LA (Carnap, 1946; 1958) constitutes its formal ontology counterpart. In QFT no microscopic physical system is conceivable as completely isolated (closed), since *it is always in interaction with the background fluctuations* (quantum vacuum condition, including in itself all the universes). In this sense, "QFT can be recognized as an *intrinsically thermal* quantum theory" (Blasone, Jizba, & Vitiello, 2011, p. ix).

Of course, because of the intrinsic character of the thermal bath, the whole QFT system can recover the classical Hamiltonian character, because of the necessity of anyway satisfying the energy balance condition of each QFT (sub-)system with its thermal bath ($\Delta E = 0$), mathematically formalized by the "algebra doubling", between an algebra and its co-algebra (Hopf algebras) (Vitiello, 2007).

The more evident difference between QM and QFT is thus the deeply different physical interpretation of the Heisenberg uncertainty principle and of the related particle-wave duality. While in QM the Heisenberg uncertainty reads:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

where $x$ is the position $p$ the momentum of the particle and h is the normalized Planck constant, in QFT the same relation reads:

$$\Delta n \Delta \varphi \geq \hbar$$

Where $n$ is the number of quanta of the force field, and $\varphi$ is the field phase. If ($\Delta n = 0$), $\varphi$ is undefined so that it makes sense to neglect the waveform aspect in favor of the individual, particle-like behavior. On the contrary if ($\Delta \varphi = 0$), $n$ is undefined because an extremely high number of quanta are oscillating together according to a well-defined phase, i.e., within a given coherence domain. In this way, it would be nonsensical to describe the phenomenon in terms of individual particle behavior, since the collective modes of the force field prevail.

In QM the uncertainty and hence the wave-particle duality relationship is between two representations, particle-like and wave-like, and accordingly the uncertainty is, respectively, on the momentum or on the position of the particle. In any case, the Schrödinger wave function in QM is not the expression of some dynamic entity like a force field, but simply the expression of different way of representing the quantum phenomenon. Hence, a conceptualist approach like Cocchiarella's Conceptual Natural Realism (CNR) to the formal ontology of QM makes sense, just as a conceptualist ontology like the Kantian one makes sense for the Newtonian classical mechanics.

On the contrary, in QFT the duality is between two dynamic entities: the fundamental force field and the associated quantum particles that are simply the quanta of the associated field, different for different types of particles. In such a way, the quantum entanglement does not imply any relationship between particles like in QM, but simply it is an expression of the unitary character of a force field. In other terms, Schrödinger wave function of QM is only a rough statistical coverage of a finest structure of the dynamic nature of reality.

### *1.3 Quantum systems as "open systems" in QFT and the notion of information*

Therefore, because of the intrinsic openness to the quantum vacuum fluctuations of any QFT system, and because of the

associated thermal bath, it is possible to define in QFT, thermodynamic operators such as "entropy" and "free energy", as well as the dynamic role they play in the different QFT systems. From the ontological standpoint, the notion of dynamically generated long-range correlations, mediated by different condensations of Nambu-Goldstone bosons, and the related notion of phase transition in terms of the dynamic constitution of different *phase coherence domains,* like as many SSB conditions of the quantum vacuum ground state, gives a new light to the Schrödinger notion of information as *negentropy* in fundamental physics.

Indeed, at the relativistic microscopic level, a phase coherence propagate with a phase velocity of the order $c^2/v$, where $c$ is the light velocity, and $v \ll c$ is the velocity of propagation of the material signal. Therefore, the dynamic constitution of a coherence domain, by the SSB of the quantum vacuum (=long-distance correlations) in the ground state, corresponds to the definition of an optimal *dynamic channeling* for the successive propagation of the energy added to the system from the thermal bath. Each SSB, indeed, because "freeing" some energy from the quantum vacuum reservoir, effectively "splits" locally it in the pair system-thermal bath, "entangled" in one only phase coherence domain, formalized as the "doubling" of the states of one only Hilbert space, extended to include also the thermal bath.

All this emphasizes the logical and ontological relevance of the following passage, synthesizing the widespread applicability of QFT in the whole domain of fundamental physics, from cosmology, to the physics of condensed matter, living (Del Giudice, Doglia, Milani, & Vitiello, 1986) and neural systems included (Freeman & Vitiello, 2006; Capolupo, Freeman, & Vitiello, 2013; Basti, 2013a; 2013b). This is particularly true nowadays, after the experimental confirmation of the so-called "Higgs mechanism" in QFT, awarded in 2014 with the Nobel Prize to P. Higgs and F. Englert.

> Quantum dynamics underlies macroscopic systems exhibiting some kind of ordering, such as superconductors, ferromagnets or crystals. Even the large-scale structures in the Universe, as well as the ordering in the biological systems, appear to be the manifestation of the microscopic

> dynamics ruling the elementary components of these systems. Therefore, in our discussion of the spontaneous breakdown of symmetry and collective modes, we stress that one crucial achievement has been recognizing that quantum field dynamics is not confined to the microscopic world: crystals, ferromagnets, superconductors, etc. are macroscopic quantum systems. They are quantum systems not in the trivial sense that they are made by quantum components (like any physical system), but in the sense that their macroscopic properties, accounted for by the order parameter field, cannot be explained without recourse to the underlying quantum dynamics (Blasone, Jizba, & Vitiello, 2011,p. ix).

To sum up, what the contemporary fundamental physics and fundamental biology need is a formal ontology of the *causal foundation of the natural laws*. Namely, it is necessary a deep change of paradigm in ontology as to the LA of the Newtonian physics and of the Kantian epistemology, which imply, on the contrary, a *logical foundation of causality*, and hence an atomistic and reductionist approach to complex phenomena (Mazzocchi, 2008).

Of course, for formally justifying a causal foundation of the natural laws the "causal indexation" of modal operators (and of logical connectives), by an arbitrary cut-down of the relative quantifiers, among the non-denumerable many types allowed as in Cocchiarella's CNR, it is not the solution (Cocchiarella, 2007). Indeed, the use of the causal/natural indexation of quantifiers and modal operators in the CNR formal ontology is based only on "contingent hypotheses", since it is not possible for him to give a logical calculus for the natural realism (Cocchiarella, 2007, p. 280). On the contrary, there exists such a possibility, but it ultimately requires a coalgebraic foundation of modal logic we cannot deepen here (Blackburn, De Rijke, & Venema, 2002).

On the other hand, the attempt in fundamental physics of developing the so-called "causal-set theory" goes in the same direction, as both L. Smolin and R. Penrose also recently emphasized (Smolin, 2013; Penrose, 2013, p. xxiv; Kronheimer & Penrose, 1967). Even though it is not sufficient, because, in set theory, it is in principle impossible to deal formally with a constructive approach to arboreal

structures, without supposing the König lemma and the infinity of its paths. This is not the case, however, of the first-order coalgebraic semantics, where set-trees are always finitarily upper bounded.

Let us see now how the Natural Reaslism (NR) formal ontology can suggest an original solution of such problems because, from one side it can support a non-reductionist approach to the constitution of the different types of "wholeness", either characterizing the complexities in natural sciences, or the "sethood" in logic and mathematics. On the other side, NR is able, to support a constructive approach for demonstrating how "arboreal" structures (i.e., based on non-transitive, non-symmetric, non-reflexive relations) can justify the existence of equivalence structures (i.e., based on transitive, symmetrical and reflexive relations), like classes or sets, the logical laws ruling them included. That is, what the causal set theory is searching for, without having the formal means for solving the problem.

## 2. A semi-formal presentation of the NR formal ontology

### *2.1 Premise: the distinction between logical and causal necessity*

#### 2.1.1 Quine's criticism to C. I. Lewis' modalization of the logical implication

In the first chapter of one of his masterpieces, *Mathematical Logic,* W. V. O. Quine, rightly emphasizes the difference between the semantic sense of the term "implies", strictly related with the notion of "truth", and the syntactic sense of the logical connective "if… then" and of its symbol "⊃". Based on these considerations too often neglected, Quine criticizes, on one side, Whitehead and Russell, who blurred in their *Principia* such fundamental distinctions, and, on the other side, Lewis and Smith who, trying to solve such a misunderstanding, essentially missed the point.

What Quine rightly rebukes to C. I. Lewis' notion of "strict implication" is that it is a long way from representing a satisfactory theory of implication. At last, it offers a theory of "modes of statement composition" according to conditionals "of a not truth-functional sort"

typical of the so-called intensional, and not extensional interpretations of modal logic.

On the contrary, according to Quine, a theory of implication that is satisfactory for metaphysical uses as Lewis originally pretended, has to take "statements as names of some sort of entities", so "to take implication as a relation between those entities, rather than between the statements themselves". Finally, it must be a theory able to justify also the "difference" or the "identity" of the entities designated by these statements, since the problem concerns not only relations such as "implication", but also like "equivalence", "compatibility" and similar (Quine, 1983, pp. 31-33).

With these words, it is evident that Quine is saying us that a satisfactory theory of ontological implication has to be:

1. A theory of *metaphysical* and not *logical* implication, and hence of *causal* and not *logical* necessity, because it has to deal with relations among existing *entities* and not only among the statements referring to them.
2. A theory able to justify on a causal basis either the "differences", or the "identities" among the denoted entities.
3. A theory able to illuminate, on the same causal basis, the alleged "obscurity" of such referential entities that are, in the light of Quine's examples and of the precedent discussion, "natural kinds"[1] or, if we want to use a word banned from the modern philosophical jargon, "natural essences", or in short, "natures".
4. Finally, a theory able to give also an ontological foundation of the notion of *truth,* given the strict relation existing between the notions of "implication" and of "truth".

Let us see how NR ontology is able to satisfy all these requirements.

### 2.1.2 A Medieval suggestion

The suggestion for a possible solution of the problem is coming

[1] Even though this is not in continuity with Quine's teaching, who always criticized the notion of "natural kind" as "objects" to which the common names refer. "Natural kinds", however, both in NR and CNR, are not "objects" at all.

from the Middle Age, by Thomas Aquinas who was interested like us in the foundation of a naturalistic ontology based on the *causal necessity*, as opposed to the logical necessity of the Platonic metaphysics. His aim was, indeed, to make the Christian metaphysics and theology compatible with the emergent naturalism of the Aristotelian ontology in the newborn Universities of the beginning of the second millennium.

Our aim, at the beginning of the third millennium, is similar, not only because we share the same theological convictions, but also because this is today in continuity with the necessity of substituting the logical realism of the Newtonian physical laws (=LA) with the naturalism of the evolutionary approach in actual cosmology.

In this light, there is a fundamental passage from Aquinas *Commentary on Aristotle's Physics,* in which he explains what is proper of the demonstrative procedures in physical sciences, as based, differently from mathematical sciences, on the causal necessity, and not on the logical necessity.

In the "Lecture 15" of this *Commentary* on the Second Book of *Physics*, with the title: "How necessity is found in natural things" (see, for the English translation of this text: (Blackwell, Spath, & Thirlkel, 1999, p. 135ff.)), Aquinas comments an Aristotelian passage [*Physics*, II, 199b,34 - 200b,9] very significant for our aims. In it Aristotle starts, by asking himself "whether the necessity (in physics) is 'hypothetical' (*ex ypothéseos*) or 'simple' (*aplôs*)", that is "assertoric", like in metaphysics and in logic. Aristotle chooses the first alternative, namely, the physical demonstrations are hypothetical, because of the contingent nature of the physical beings, but with an important difference, according to the *a priori* or *a posteriori* character of the causes involved[2].

[2] According to the translation offered by Blackwell *et al.* in their version of the Aquinas *Commentary,* the core of the Aristotelian argument [*Physics*, II, 200a 15-33] reads as follows: "Necessity in mathematics is in a way similar to necessity in things which come to be through the operation of nature. Since a straight line is what it is, it is necessary that the angles of a triangle should equal two right angles. But not conversely; though if the angles are not equal to two right angles, then the straight line is not what it is either. However, in things which come to be for an

The Aquinas commentary on this passage is very interesting, because he makes a logical analysis of these two types of demonstration, showing a deep knowledge of the Stoic sentential logic of the hypothetical demonstrations, and of their two fundamental laws of the *modus ponens* and of the *modus tollens,* effectively not yet discovered by Aristotle. Moreover, this passage demonstrates as well his deep knowledge of the logic of the "converse implication", of which Aquinas suggests in this case a modal development that is original, both with respect to the Aristotelian modal syllogism (Hintikka, 1972; van Rijen, 1989; Nortmann, 2002; Malink, 2006), and with respect to Lewis' modal version of the logical implication.

Effectively, Aquinas is able to distinguish, in his Commentary of the quoted passage of Aristotle, two types of hypothetical demonstrations in physics, corresponding in terms of modern logistics, respectively, to the logic of the *direct* and of the *converse* implication. The first one, Aquinas says, is indeed similar to the "simple" character of the assertoric demonstrations of mathematics, which is the case when the demonstration is by causes that are *prior* as to the physical process, i.e. the *initial causes* from which a given process starts: the *material* and the *efficient* causes. In the LA of the modern Newtonian physics, these correspond to the *initial conditions* — respectively, the *position* and the *momentum* — of the classical mechanics.

> effect, the reverse is true. If the effect is to exist or does exist, that also which precedes it will exist or does exist; otherwise just as there, if the conclusion is not true, the premise will not be true, so here the effect or 'that for the sake of which' will not exist. For this too is itself a starting-point, but of the reasoning, not of the action; while in mathematics the starting-point is the starting-point of the reasoning only, because there is no action. If then there is to be a house, such-and-such things must be made or be there already or exist, or generally the matter relative to the end, bricks and stones if it is a house. The end, however, is not due to these except as the matter, nor will it come to exist because of them. Yet if they do not exist at all, neither will the house, or the saw—the former in the absence of stones, the latter in the absence of iron—just as in the other case the premises will not be true, if the angles of the triangle are not equal to two right angles. The necessary in nature, then, is plainly what we call by the name of matter, and the changes in it. Both causes must be stated by the physicist, but especially the end; for that is the cause of the matter, not vice versa".

Effectively, in another passage of his Commentary to Aristotle's *Physics* (*In Phys.*, II, 11, 1-9), Aquinas says that this is the case of Democritus, mechanistic approach to physics. In it, no "formal cause" is necessary, since it supposes that the *final* state of a physical process is completely determined by its material and efficient, *initial* causes. In this way, he states, the initial causes are like the postulates of a geometric demonstration, while the final state is like a theorem deduced by such postulates, so that physics demonstrations are at all similar to geometry demonstrations.

However, Aquinas annotates, we cannot apply this logic to the *generation processes* of a new accidental or substantial form in the matter substratum, which correspond to what in physics we denote as *phase transitions*. Namely, the generation of a new accidental form occurs when a new state of a given substance occurs, without changing its nature (e.g., the passage of water between the icy and the liquid state, in both senses). On the contrary, the generation of a new substantial form occurs when the transformation changes the nature of the substance (e.g., like in chemical reactions, and/or in biological generation of new individuals). In both cases, indeed, the initial causes are not able at all to determine completely the final state of the dynamics, so to make it fully predictable starting from them. We have thus to consider the physical process as a whole, the final state included. In such cases, indeed, the *formal causality* is involved[3].

To sum up, in this case of the form-generation processes, Thomas continues, the logic of this type of inference is like when we reason about something that "ought to be" (*debeat esse*), as far as this is the intended final state of a given action by an intentional agent, so to be in its *active power*. With a fundamental difference, however. In the case of a physical process of a form generation, there is no *intentionality* involved, and so *no finalism*, so that we are not in the semantic realm of the *deontic* logic, but in the realm of the *alethic* logic, even though the syntax of the two forms of demonstration is the

[3] Recently, the German philosopher U. Meixner developed a semi-formal analysis of the "formal necessity" in the Aristotelian theory of causality similar to ours, even though from the standpoint of the ontology of the logical realism (Plato), and not of natural realism (Aristotle) like ours (Meixner, 2003).

same. Practically, it is like in our case, where we use the syntax of **KD45** ML not in a deontic intensional interpretation, but in an alethic (ontological) intensional interpretation.

What Aquinas is suggesting us, indeed, is that we are only reckoning with *a non-mechanistic, but dual ontology of the physical causality,* because involving not only a change in *matter* but also in the *form* of the process – the coherence domain of a phase transition, in QFT terms. That is, a physical causality where the end-state of the physical process does not depend only on the initial conditions, like in the Newtonian (and Democritean) mechanics, because the initial components of the process lose their individuality, so that *a new collective behavior of the system emerges.* Hence a *new natural form* emerges, even though Aquinas does not use the term "emergence", but the term "eduction" (*eductio*) of the new form, evidently as opposed to the term "deduction", valid for the "geometrical" logic of demonstrations in Democritus' (and Newton's) mechanics[4].

In both cases – the *intentional* case and the *physical* case of the form emergence –, Aquinas says, it is like if the final state – either if it is intended by some intentional agent (psychology), or if it is not (physics) –, plays the role of the premise of the inference. This means that formally, we are faced with a *converse implication,* $<p \leftarrow q>$ and no longer with a *direct implication* $<p \rightarrow q>$.

To sum up, Aquinas is affirming here an *analogy* – formally: the same *syntactic* structure, interpreted semantically onto two different *domains* – between *the causal realm of ontology*, when we deal with a physical process, in which a new level of matter organization emerges, as unpredictable from the initial material and efficient causes, and *the deontic realm of intentionality*. Without confusing them, however, that is, without confusing the "to be" (alethic), with the "ought to be" (deontic) at the semantic level, so to fit perfectly with the requirements of the so-called "Hume law". In both cases, indeed, their logic, their syntactic structure, is the logic of the *converse implication,*

[4] It is evident the similarity with the ontology underlying QFT and its interpretation of the particle-field duality principle, we discussed in §1.2 $<\Delta n \Delta \varphi \geq \hbar>$, whereas the collective modes of the force field prevail over the individuality of the particles.

according to the **KD45** ML, interpreted as a **Secondary S5**, and not as a **Deontic S5,** following the two more diffused, ordinary ways of connoting **KD45** in current modal logic literature.

Aquinas states explicitly all this in a successive passage that we report here completely:

> Next where he [Aristotle] says, 'Necessity in mathematics ...' (200 a 15), he compares the necessity which is in the generation of natural things to the necessity which is in the demonstrative sciences. (…)
>
> Indeed, an 'a priori' necessity is found in the demonstrative sciences, as when we say that since the definition of a right angle is such, it is necessary that a triangle be such and so, i.e., that it have three angles equal to two right angles. Therefore, from that which is first assumed as a principle, the conclusion arises by necessity [i.e., the logical necessity of the *modus ponens*].
>
> The converse, however, does not follow, i.e., if the conclusion is, then the principle is. Because, *sometimes*, a true conclusion can be drawn from false propositions. On the contrary, it does follow that if the conclusion is not true, then, neither is the given premise true. Because, a false conclusion can be drawn only from a false premise [i.e., the logical necessity of the *modus tollens*].
>
> On the contrary, in things which happen for the sake of something (*quae fiunt propter aliquid*), either according to technique, or according to nature, this converse does obtain [i.e., according to the connective of the *converse implication*]. For, if the final state (*finis*) either will be or is, then *it is necessary* that what is prior to the final state either will have been, or is [i.e., it is not question of time][5]. If, however, that which is prior to the final state is not, then the final state will not be, just as in demonstrative sciences, if the conclusion is not true, the premise will not be true [i.e., both in direct and converse implication if the antecedent is false, the consequent is false too].
>
> It is clear, therefore, that in things that come to be for the sake of a final state, the final state holds the same order that the premise holds in demonstrative sciences. This is so because the final state also is a

[5] Namely, by using the modal operator "it is necessary", what Aquinas is here suggesting is the necessity of a modalization of the converse implication for being the proper modal logic of the causal implication. See below for explanation.

> principle, not indeed of action, but of reasoning. For, from the conclusion we begin to reason about those things that are the means for reaching such a conclusion. In demonstrative sciences, however, we do not consider a principle of action, but only a principle of reasoning, because there are no actions in demonstrative sciences, but only demonstrations. Hence, in things that happen because of reaching a final state, this properly holds the place that the premise holds in demonstrative sciences. Hence, there is a similarity on both sides, even though they seem related conversely because of the fact that the end is last in action, which does not pertain to demonstration (Aquinas, *In Phys.*, II, 15, 5) [Square parentheses are mine].

Aquinas suggestion is thus double:

1) The logic of the emergent complexities in physics (form generation), and/or of the spontaneous symmetry breakdown of the infinitely many quantum vacuum conditions in QFT, is the logic of the converse implication, i.e., the logic of the *causal necessity* (= formal causality) as irreducible to the logic of the *logical necessity*;
2) If we want to have a proper formal ontology of the causal necessity, as far as – against Leibniz posit – it is not reducible to the logical necessity, we need to give *a modal version of the converse implication* as the proper logic of the *causal entailment.*

In other terms, just as the modal version of the material implication, i.e., the so-called "strict implication" of C. I. Lewis gives a definition of the *logical entailment*, i.e., "*q* follows logically from *p*" (Huges & Cresswell, 1996, p. 203)[6], the opposite holds for the *causal entailment*, i.e., "*p* precedes causally *q*". This is the proper logic of the formal ontology of NR, as opposed to the LA of clas- sical mechanics, or as opposed to the CNR of QM.

[6] As they explain, "to say that a proposition, *p,* entails a proposition, *q,* is simply an alternative way of saying that *q* follows logically from *p,* or that the inference from *p* to *q* is logically valid".

## *2.2 The logic of the converse implication as the logic of the causal necessity*[7]

To sum up, the double, convergent suggestions from Aquinas and Quine invite us to a deep re-consideration of the axiomatic theory of modal logic inherited by the pioneering work of C. I. Lewis at the beginning of the last century.

As we know, C. I. Lewis defined the notion of strict implication for avoiding the well-known paradoxes of implication related to the notion of the truth-functional conditional "if-then", interpreted as material *implication* of the mathematical logic[8]. I.e., given the truth table of the material implication:

| | $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|---|
| 1. | 1 | 1 | 1 |
| 2. | 1 | 0 | 0 |
| 3. | 0 | 1 | 1 |
| 4. | 0 | 0 | 1 |

Several paradoxes, the so-called "paradoxes of the material implication" (Huges & Cresswell, 1996, p. 194), follow from this truth table, such as:

1. $p \rightarrow (q \rightarrow p)$
2. $\neg p \rightarrow (p \rightarrow q)$

I.e.: (1) given a true proposition, any proposition, either true or false, can imply it; (2) if a proposition is false, it implies any proposition whatsoever. Moreover, since for any proposition *p,* either the antecedent of (1), or the antecedent of (2) must be true, also the following paradox holds:

3. $(p \rightarrow q) \vee (q \rightarrow p)$.

[7] For this sub-section and the following one I am deeply indebted with the doctoral work (Panizzoli, 2013) of my former student and now my Assistant Dr. Francesco Panizzoli.

[8] See the deep reflections of Quine on this regard, summarized in §2.1.1.

For avoiding such paradoxes it is sufficient, Lewis suggests, to make "stronger" the notion of "implication"*,* so to distinguish between implications that hold materially, and implications that hold *necessarily* or *strictly,* namely, it is necessary that if *p* is true, so is *q.* From this the definition of the "strict implication" (↠) follows:

$$\textbf{Def.1}: (\alpha \twoheadrightarrow \beta) := \left(\Box(\alpha \rightarrow \beta)\right) \Leftrightarrow \left(\neg\Diamond(\alpha \wedge \neg\beta)\right)$$

Where $\alpha$ and $\beta$ are propositional meta-variables.

Practically, it is like if we eliminate from the truth table of the material implication the 2nd row, so to grant the fundamental law of logical semantics that *truth is always preserved in any valid inference*, that is:

| | *p* | *q* | *p*↠*q* |
|---|---|---|---|
| 1. | 1 | 1 | 1 |
| 2. | 1 | 0 | 0 |
| 3. | 0 | 1 | 1 |
| 4. | 0 | 0 | 1 |

The intrinsic relationship between logical semantics and strict implication forces us to interpret the strict implication as "entailment", that is, as a relation between *true propositions* and not between wff's. Namely, <*p* ↠ *q*> properly means "*p* entails *q*" as the converse of "*q* follows *logically* from *p*", or, in other terms, it properly means that "the inference from *p* to *q* is *logically* valid". This semantics, however, originates the so-called "paradoxes of the strict implication". They, unfortunately, are as many very strong ways for asserting that the so-called "principle of Pseudo-Scotus" or the "principle of explosion" (EP) (*ex contradictione sequitur quodlibet*) is a valid inference in logic (see paradox (1) below). According to (Huges & Cresswell, 1996, p. 203), a list of such paradoxes is, indeed, the following:

1. $(p \wedge \neg p) \twoheadrightarrow q$
2. $q \twoheadrightarrow (p \vee \neg p)$
3. $\neg\Diamond p \rightarrow (p \twoheadrightarrow q)$
4. $\Box q \rightarrow (p \twoheadrightarrow q)$

Now, Lewis himself stated that, if we want to avoid (1) and the other related paradoxes, we have to exclude other intuitively valid principles, before all the so-called "principle of the disjunctive syllogism":

$$\big((p \vee q) \wedge \neg p\big) \rightarrow q$$

However, for excluding this principle, it is necessary to refer to the so-called *relevance logics* (Huges & Cresswell, 1996, p. 205), i.e., it is necessary to define a valid criterion of *relevance* of a premise as to a given conclusion, that means using the notion of *paraconsistent* negation, refusing the general validity of the *same extensionality* between a proposition and its negation (Béziau, 2000). After our semi-formal presentation of the NR formal ontology we see that the logic of NR is precisely a *relevance logic,* introducing a *formal* ontological criterion of relevance of a given premise as to a given conclusion.

As a first step, following Aquinas suggestion, let us introduce now the notion of *converse implication* and of its "strict", modal version. The truth table of the converse implication is the following:

| | $p$ | $q$ | $p \leftarrow q$ |
|---|---|---|---|
| 1. | 1 | 1 | 1 |
| 2. | 1 | 0 | 1 |
| 3. | 0 | 1 | 0 |
| 4. | 0 | 0 | 1 |

Anyway, if we interpret the converse relation as a *syntactic* relation among wff, it has no relevance for an ontology that, as such, is simply an interpretation of a modal calculus. On the contrary, if we want to use the converse implication for justifying a formal ontology of the *causal* necessity as complementary of the *logical* necessity, we have to interpret also it semantically, as a *strict* converse implication relating statements denoting *things causally related,* as Quine required for justifying a notion of *ontological implication* (see above 0). In such a case, it makes sense to define the notion of *causal* necessity, as eliminating the possibility that an *effect* (denoted by *q*) *exists* without its

*cause* (denoted by $p$) exists[9]. In other term, we have to eliminate the 3rd row of the converse implication truth table. I.e.:

| | $p$ | $q$ | $p \nleftarrow q$ |
|---|---|---|---|
| 1. | 1 | 1 | 1 |
| 2. | 1 | 0 | 1 |
| 3. | 0 | 1 | 0 |
| 4. | 0 | 0 | 1 |

From this truth table, the semantic interpretation of the "strict converse implication" ($p \nleftarrow q$) derives, with the meaning "$q$ entails $p$", i.e., ontologically, "(the effect connoted by) $q$ entails (its cause connoted by) $p$", which is the converse of "*p precedes* causally $q$". This reading of an *ontological entailment* is the opposite of "*q follows* logically from $p$", expressing the *logical entailment* of C. I. Lewis' strict implication just discussed, because of the reversal of the connective between the causal (ontic) and the logical realm. In this way, we can write the definition of the strict converse implication, as the key-notion of the logic of the causal necessity, $\Box^C$.

$$\textbf{Def.2}: (\alpha \nleftarrow \beta) := \left(\Box^C(\alpha \leftarrow \beta)\right) \Leftrightarrow \left(\neg\Diamond^C(\neg\alpha \wedge \beta)\right)$$

Because of the relationship between implication and inclusion, and because, in this case, the necessity condition is given in the antecedent of the conditional, we can define the notion of the *causal inclusion* ($p \supseteq_C q$) as complementary of the usual *logical inclusion* ($p \subseteq q$). Consequently, the semantic notion of "*p precedes* causally $q$", or,

[9] It is evident that we must interpret both $p$ and $q$ as denoting as many *events/beings* in the ontological causal-effect relation between them. In other terms, they correspond semantically to "definite descriptions" of the respective referents. We see thereafter that in NR it is possible to justify a theory of the *direct, causal* reference, not between names and things, but between categorical propositions, like $p$ and $q$, and their referents, based on a *natural* and not *conceptual* theory of the "double saturation" between the unary predicate and its only argument of each definite descriptions.

shortly, "$p$ causes $q$", is the ontological interpretation of the strict converse implication. That is, $(p \rightarrow_C q)$ is the *ontological* counterpart in the natural realm of the *semantical* reading of $(p \twoheadleftarrow q)$, as "the effect (connoted by) $q$ entails its cause (connoted by) $p$" in the logical realm. This is the inversion of the direction of the inference between the *ordo essendi* and *ordo cognoscendi* ("what is first in being, is last in knowing") of the Aristotelian epistemology. We met already with it in Aquinas passage on the converse implication quoted in §2.1.1, and we discuss again in §0 about this inversion, for "lifting the fog on complexity" about the related and very ambiguous notions of "downward" and "backward" causation.

Of course, the collection of objects included in the domain of the same causal relation does not constitute properly *a class*, but a *natural kind*. In this category, no class membership predicate <∈> holds, but the converse notion of "co-membership" <∋>, in the sense that a genus "admits" (and not simply "includes") several different species, by a "branching" of the inclusion relation[10].

Because of the strict or "intrinsic" relationship between the notion of "implication" and the notion of "truth", both on the ontological and on the logical sides, we can define on this basis an *ontological* and not *logical* condition of membership to the Universal Class **V.** We can suppose, indeed, that through a common dependence (causal inclusion) on a causal relation – effectively an "ontological entailment" – of each element of the Universal Class **V** with one only "primary generator" <Γ>, many "secondary" transitive-antisymmetric-

[10]This notion of "co-membership" (∋) can be formalized in a *first-order coalgebraic semantics for Boolean Algebras,* as "categorically dual" as to the notion of "membership" (∈) in a Boolean logic (Venema 2007; Basti 2015). The symbol "∋" properly means "admittance" and not simply "inclusion", in the sense that a coalgebraic modal semantics is defined on "non-wellfounded (NWF) sets". That is, sets $x$ in which "self-inclusion" is admitted, i.e., $\{x\}$= "singleton", and therefore infinite unbounded chains of set inclusions are allowed, so that no set *total ordering* is possible, but only infinitely many *partial* ones (Aczel 1988). In this way, the direct inclusion relation from a superset ("ascendant") might follow different "branching" in the set inclusion trees, so that subsets ("descendants") are properly "admitted" not uniquely "included". It is possible then to define in coalgebras the notion of "equivalence by bisimilarity"⇆ (or "bisimulation").

reflexive (partial ordering) relations among these dependent elements could be constituted, and hence, equivalence domains among them.

In this way, not only the *necessary*, but also the *sufficient* condition for the full membership to **V** – and hence for the "full (actual) existence" of each of its member – is given, according to a proper *Ontological Axiom of Existence* (**OAE**) of such a formal ontology, the NR formal ontology, as we see.

### *2.3 The NR formal ontology*

We can now give a semi-formal, synthetic presentation of the NR formal ontology and of its logic:

VARIABLES

$x, y, \ldots$: individual variables
$\alpha, \beta, \ldots$: individual meta-variables
$P, Q, \ldots$: predicate variables
$\xi, \zeta, \ldots$: predicate meta-variables

CONSTANTS

$a, b, \ldots$: individual constants
$\mu, \nu, \ldots$: individual meta-constants
$P, Q, \ldots$: predicate constants
$\varphi, \psi, \ldots$: predicate meta-constants
$E!(a)\ldots$: existence predicate for individuals $a$
$\Gamma$: primary generator (**PG**)
$\gamma$: secondary generator (**SG**)

COLLECTIONS

$\overset{\subset}{\mathbf{V}}$ : universal collection [including only individuals, potentially existing]
$\mathbf{V}$ : universal class
$\mathbf{\Lambda}$ : empty class
$A, B, C, \ldots$: natural kinds (genera, species…)
**A, B, C, …**: logical classes

CONNECTIVES

$\neg, \wedge, \rightarrow, \leftarrow, \leftrightarrow$ : propositional connectives

*non, et,* $\Rightarrow, \Leftarrow, \Leftrightarrow$ : propositional meta-connective

$\leftrightarrows$: coalgebraic equivalence by bisimilarity (note 10)

$\vDash (p \twoheadrightarrow q)$ : logical entailment, i.e., "*p* entails *q*", or "*q* follows *logically* from *p*", or "the inference from *p* to *q* is *logically* valid"

$\vDash (p \twoheadleftarrow q) = (p \rightarrow_c q)$: causal entailment, i.e., "*q* entails *p*", or "*p* precedes *causally q*", or "the inference from *p* to *q* is *causally (ontologically)* valid"

$(p \supset_C q)$ *vs.* $(p \subseteq q)$ : causal *vs.* logical inclusion, i.e., "*p* admits ($\ni$) causally *q*" *vs.* "*q* includes logically *p*" (see note 10).

QUANTIFICATION

$\forall, \exists$ : binding variables denoting what potentially is

$\forall^e, \exists^e$: binding variables denoting what actually exists

$\forall^m, \exists^m$: binding variables denoting what mentally exists (concepts)

$\hat{x}...$: the collection/class of *x* such that…

MODAL AXIOMS OF NR (and correspondent first-order formulas in NWF set-theoretic semantics. See note 10 and (Venema 2007))

**N**: <(**X**→α) ⇒ (□**X**→□α)>

**K**: <**k** + **N**> where **k** is the propositional calculus.

**T:** <□α→α>$(\forall x) R(x,x)$: *R* is reflexive

**D:** <□α→◊α>$(\forall x \exists y) R(x,y)$: *R* is serial

**4:** <□α→□□α> $(\forall x,y,z)((Rx,y) \wedge (Ry,z)) \rightarrow (Rx,z)$: *R* is transitive

**5** or **E**: <◊α→□◊α>
$(\forall x,y,z)((Rx,y) \wedge (Rx,z)) \rightarrow (Ry,z)$: *R* is Euclidean

**KD45 or Secondary S5** system is the ML of NR.

ONTOLOGICAL AXIOMS OF EXISTENCE (**OAE$_{1\text{-}2}$**) FOR NR

$\forall x\left(x \subset \overset{\subset}{\mathbf{V}}\right) \leftrightarrow \hat{x}\left(\Gamma \rightarrow_C \{x\}\right)$: Principle of causal inclusion of individuals

(or ontological "singletons", see n. 10) in the c*ollection* $\overset{\subset}{\mathbf{V}}$ **,** i.e.:

$$\left(\mathbf{OAE_1}\right): \quad \forall x \begin{cases} \text{either } \left(\left(\left(\Gamma \rightarrow_C \{x\}\right) = 1\right) \rightarrow x \subset \overset{\subset}{\mathbf{V}}\right) \leftrightarrow \exists x \\ \text{or } \left(\left(\left(\Gamma \rightarrow_C \{x\}\right) = 0\right) \rightarrow x \not\subset \overset{\subset}{\mathbf{V}}\right) \leftrightarrow \neg\exists x \end{cases}$$

Where Γ is the "Primary Generator" (**PG**) or "Primary Cause of everything", defined as following:

**Def. 3. (PG):** $\Gamma \coloneqq \left(\left(\Gamma \supset_C \overset{\subset}{\mathbf{V}}\right) \wedge \neg\left(\Gamma \subseteq \overset{\subset}{\mathbf{V}}\right)\right)$

Definition of the Principle of Primary Causation **PC:**

**Def. 4.** (**PC**): $\overset{\rightarrow_C}{R_\Gamma} \coloneqq \forall x\left(\Gamma \rightarrow_c \{x\}\right)$ i.e., "Everything exists potentially in the active power of Γ"

For justifying the existence of differences among the elements of $\overset{\subset}{\mathbf{V}}$ and hence the self-identity of each of them, so to grant their membership to the universal class **V** it is necessary to demonstrate a Lemma of Secondary Causation (**LSC**), namely:

$$\mathbf{LSC}: \forall x, y \left[\begin{array}{l} \left(\left(\Gamma \rightarrow_C \{x\}\right) \wedge \left(\Gamma \rightarrow_C \{y\}\right)\right) \wedge \\ \wedge\left(\left((y \neq x) \Rightarrow (x \rightarrow_C y)\right) \vee \left((y = x) \Rightarrow \neg(x \rightarrow_C y)\right)\right) \end{array}\right]$$

*Demonstration:* It follows immediately from **OAE$_1$** and from the Euclidean rule of "weak transitivity", applied to the relation ($\rightarrow_C$) and to its anti-symmetric application, instead of the ordinary transitive rule, given that no total ordering, and only set inclusion trees are allowed in NWF sets (see n.10).

> **Remark 1:** On this regard, we answer by **LSC** the deep notation of Quine quoted in §2.1. Over there, he said that a satisfactory *ontological* theory of implication would require the

rigorous definition of the conditions according to which the entities designated by the two statements, put into the relation of ontological implication, can be said "to be the same or different entities". In parentheses, **OAE$_1$** gives us also a first "enlightenment" onto the "obscurity" of such entities. They are simply the beings, which "were", "is", or "will be" existing in the universe(s), as far as their common *existence* is the product of a fundamental or "primary" causality embracing the whole history of the universe(s), and as far as their different *species* are the product of "secondary" causes, at different stages of (each) universe history.

**Remark 2:** In other words, any difference in NR ontology has a *causal* justification, by a "texture" of secondary causes *inside* a given universe[11], not outside it. Such a principle is perfectly fitting with the "energetically open" character of any quantum system in QFT, within an "energetically closed" universe[12].

[11] This formal ontology fits with the Aristotelian one, where "qualities" are properly "actions/passions" (*actiones/passiones*), e.g. "being white" is properly a "whitening", "being black" is properly a "blackening", etc., just as, conversely, in logic, any predicate verb can be translated into its participial form plus the copula "is" (e.g., "Mark loves" corresponds to "Mark is loving", etc.). More deeply, all the "sensible qualities" (e.g., colors) are only the causal effects on the five senses of the four, more fundamental "active/passive" qualities. They are effectively, four fundamental dynamical forces, from which all the other "qualities" (forces, *virtutes* in Latin) derive, all related with heat ("hot" and "cold", active; "humid" and "dry", passive), by which the four elements ("water", "earth" , "air", and "fire") interact (thermo-)dynamically among them, for the ultimate material constitution of all physical bodies. Indeed, the "water" is "cold/humid", the "earth" is "cold/dry", the "air" is "hot/humid" and the "fire" is "hot/dry". For this reason, a sufficient amount of water can extinguish, for instance, the fire, or, conversely, an insufficient amount of water makes it transformed by fire into air (vapor).

[12] For this reason the causal action from the first generator <Γ> cannot be interpreted at all as the ultimate "energy reservoir" of the universe, like if it was a "boundary condition" for the universe dynamics. This is, for instance, the mistake of Descartes' metaphysical theology. This is not, however, the mistake of Aquinas' one, as erroneously Hawking states in many passages of his last book on the ontology of cosmology (Hawking & Mlodinow, 2010). In NR, the collection $\overset{\subset}{\mathbf{V}}$ plays the role of the quantum vacuum, in which everything potentially exists (§1.2).

So, on the basis of **LSC,** we can define the notion of secondary generator (**SG**), <γ>:

$$\mathbf{Def.5(SG)}: \gamma := (\exists \gamma)(\forall x, y, z)\big((x \to_c y) \wedge (x \to_c z)\big) \wedge \wedge \left( \begin{array}{l} \big(((y \to_c z) \wedge (z \to_c y)) \to (y = z)\big) \vee \\ \vee \big(((y \to_c z) \wedge \neg(z \to_c y)) \to (y \neq z)\big) \end{array} \right) \wedge (x = \gamma)$$

We can therefore introduce the notion of "secondary causation", **SC**, as the causal relation $\to_C$ relative to a given $\gamma$, i.e.:

$$(\mathbf{SC})\ \vec{R}_\gamma := \big(\forall x, y(x = y) \Rightarrow (\gamma \to_c (x, y))\big) \wedge \big(\forall w, z(w \neq z) \Rightarrow ((\gamma' \to_c w) \wedge (\gamma'' \to_c z))\big)$$

I.e., "any identity/difference has a causal justification in (is in the causal power of) its proper generator <γ>". As **SG** emphasizes, $< R_\gamma >$ is nested inside $< R_\Gamma >$. Namely, the causal power on the *natures* (essences, genera/species) of the different things $< \vec{R}_\gamma >$.

Formally, this depends on the fact that **SG,** as far as defined on NWF sets in which set self-inclusions are allowed (see n.10), defines effectively a *partial ordering* (i.e., a reflexive, transitive, and anti-symmetric relation) among sets ordered by inclusion, where the transitive rule is the "weaker" Eucidean one.

In ontological terms, the "secondary generator(s)" is (are) element(s) of the collection $\overset{\subset}{\mathbf{V}}$ acting causally on other elements of $\overset{\subset}{\mathbf{V}}$, by which the differences/identities among them ultimately depend. In such a way, it is possible to distinguish between a complementary, double, *composite* relation of "causal entailment" from <Γ> and <γ>, i.e. $\left(\vec{R}_\Gamma \circ \vec{R}_\gamma\right)$– or, more simply, the related functional scheme: $<(\Gamma \circ \gamma) \to_c \_\_>$ – defining, respectively, the *necessary* and the *sufficient* condition for the *membership* to the universal *class* **V,** and hence for the *actual existence* of whichever thing

Indeed, based on **SG** and **SC**, we can define also the notion of *ontological self-identity*, **OSI,** that is of "causally founded self-identity" as a formal version of the "being for itself and in itself" of any individual substance or "subsistent being" of classical metaphysics. Namely, instead of having like in classical logic:

$$(x=x) := (\forall x, w)(x \in w) \leftrightarrow (x \in w)$$

We have:

$$\mathbf{OSI}: (x=x) := (\forall^e x)((\Gamma \circ \gamma) \rightarrow_c x) \leftrightarrow ((\Gamma \circ \gamma) \rightarrow_c x)$$

So that:

$\forall^e x(x \in \mathbf{V}) \leftrightarrow \hat{x}((\Gamma \circ \gamma) \rightarrow_c x)$, principle of membership to the universal class **V,** i.e.:

$$\mathbf{OAE_2}: \forall^e x \begin{cases} \text{either } ((((\Gamma \circ \gamma) \rightarrow_c x) = 1) \rightarrow x \in \mathbf{V}) \leftrightarrow \exists^e x \\ \text{or } (((\Gamma \rightarrow_c x) = 0) \rightarrow \neg x \subset \Lambda) \leftrightarrow \neg \exists x \end{cases}$$

In such a way, the existence of both **V** and of its complement **Λ** is granted, since, in the causal entailment $<(p \rightarrow_c q)>$, for $p \equiv 1$, both $q \equiv 1$ and $q \equiv 0$ are allowed (see §2.2).

Two consequences deriving immediately from **OAE$_{1\text{-}2}$** double existence axiom, are:

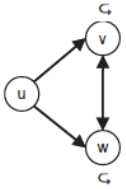
**C1:** NR gives an ontological and not conceptual like in CNR, justification of the difference between the *possibilist* $<\forall/\exists>$ and the *actualist* quantifiers $<\forall^e/\exists^e>$.

**C2:** NR introduces with the quantifiers, also the connected modal causal operators $<\Box^c/\Diamond^c>$, not through arbitrary Henkin-like "cuts" based on "hypotheses" like in CNR, but through a given set of axioms, rules and definitions, on which an ontological modal calculus of propositions and predicates can be founded.

*2.4 ML of the propositional calculus of NR*

If the introduction of the causal necessity operator $\Box_n^C$ grants that we can assume in NR logic the modal axiom **K**, we can assume also that the causal relation *C*, both primary and secondary, satisfies an *Euclidean* (not transitive!) relation between the nested generators <Γ, γ> and the elements of **V** they progressively constitute at the steps *n*. In such a way, the modal, Euclidean, axiom **5** holds too. Of course, we can assume also a *serial relation*, so that also the modal axiom **D** holds, and, finally, *secondary reflexive* and *transitive* relations, hold exclusively among the elements of **V** (without including the primary generator <Γ>). In this way, the modal axiom **4** holds for the elements of **V,** all connected among them through their common relation with the primary generator <Γ>. Therefore, several *secondary* transitive-symmetrical-reflexive accessibility relations *R* are progressively admitted among the elements of **V** (with the exclusion of <Γ>), so that $\overset{\subset}{\mathbf{V}}$ constitutes a proper class generated from the starting point of **V** depending on the only <Γ>, and then, at each further step *n* of the consequent "object generation procedure", by the concourse of several nested < $\gamma$'s >.

To sum up, in the relational semantics of an over-simplified universe{**W**} with only three worlds {*u,v,w*}, and an accessibility relation *R,* where the world *u* represents the first generator <Γ>, and the subset {*v,w*}, represents the elements of **V,** considered identical because no nested $\gamma$ here occurs, we have the following. For the Euclidean relation $<\forall u,v,w\ (u\mathrm{R}v \wedge u\mathrm{R}w) \rightarrow v\mathrm{R}w>$; then, for seriality, $<\forall u,v\ (u\mathrm{R}v \rightarrow v\mathrm{R}v>$. Therefore, finally, we have:
$<\forall u,v,w\ (u\mathrm{R}v \wedge u\mathrm{R}w) \rightarrow (v\mathrm{R}w \wedge w\mathrm{R}v \wedge v\mathrm{R}v \wedge w\mathrm{R}w)>$, that is,




We can then state that **KD45**, **or secondary S5,** represented in the above Kripkean graph, is the ML of the NR formal ontology.

*2.5 QML of the predicate calculus of NR*

The QML of NR is a first-order predicate calculus, in which only *local truths* are allowed, characterizing a modal relational semantics constituted by a nested structure of possible worlds, at different levels of growing complexity (Goranko & Otto, 2007).

Effectively, several other domains and sub-domains, among the elements of **V** can be progressively "causally unfolded" by a nested structure of generation of *natural kinds* of physical things, formally corresponding to a procedure of *iterating KD45 modality* through nested *world-stories* – set-theoretically definable as nested NWF set-inclusion trees (see n. 10). Effectively, **SG** constitutes the definition of the *causal operator* of such a nested unfolding procedure, starting from the fundamental level 0. Of course, because of **OAE2**, each level $n \geq 0$ constitutes a new level of actualization of the whole universe **W**, each actualizing progressively at level *n* the causal power of $R_{\Gamma \circ \gamma_{n-1}}$, synthetically <*R**>, into a subset of actually existing objects.

Such a generation procedure is a process of worlds/objects *unraveling,* by which new sets of equivalence relations among worlds/objects – and hence a new predicative sub-domains of **V** – is *ontologically stipulated* via <*R**>, as the actual outcome of a given "world-story". *Generally,* the unraveling procedure, **UP,** in Kripke's pointed relational semantics (model theory included), where each point represents a set, is defined as follows (Blackburn, De Rijke, & Venema, 2002, p. 218):

> **UP:** Let $(W, R)$ be a Kripke's frame generated by some point $w \in W$, where $\{W\}$ is some sub-set of the whole universe **W,** at some step *k* of its evolution. The unraveling of $(W, R)$ around *w* is the frame $\left(\vec{W}, \vec{R}\right)$ where:
>
> (I) $\vec{W}$ is the set of all finite sequences $\left(w, w_1, ..., w_n\right)$, and $Rww_1,\ldots,w_{n-1}w_n$;
>
> (II) If $\vec{s}_1, \vec{s}_2 \in \vec{W}$, then $\vec{R}\vec{s}_1\vec{s}_2$, if there is some $v \in W$ such that $\vec{s}_1 + (v) = \vec{s}_2$, where + denotes a sequence concatenation.
>
> If $\mathfrak{M} = \left(W, R, V\right)$ is a model and $\left(\vec{W}, \vec{R}\right)$ is the unraveling of $(W, R)$

around *w,* and *p* is a propositional variable, then we define the valuation $\vec{V}$ on $\left(\vec{W},\vec{R}\right)$ as follows:

$$\vec{V}(p)=\left\{\left(w,w_1,...,w_n\right)\in\vec{W}\mid w_n\in V(p)\right\}$$

The model $\vec{\mathfrak{M}}=\left(\vec{W},\vec{R},\vec{V}\right)$ is called the *unraveling* of $\mathfrak{M}$ around *w*.

By unraveling, any set of formulas is thus satisfiable on an *irreflexive, intransitive* and *asymmetric* tree.

At this point we can define the notion of the "Ontological Generation Procedure", **OGP,** by which the composite relation *R** might define suitable world-stories, by unraveling (making actual) new *domains* of worlds/objects, potentially existing in the causal power of the preceding secondary generator(s), all included (causally entailed) in the causal power of the first generator.

**Remark:** We could define **OGP** also as an "ontological *stipulation* procedure", so to justify S. Kripke's remark that in modal relational semantics possible worlds and objects are to be *stipulated* not "observed" as already given before us. That is, they are to be "posed on the basis of axioms and rules", and never "arbitrarily supposed" as given in some Platonic realm. Indeed, the notion of "ontological stipulation" in NR has to be intended like when we pose in axiomatic geometry new possible objects, because satisfying the geometry rules and axioms, like when we say, in a geometrical valid demonstration: "let us suppose that there exist the triangle *ABC*…". Similarly, in NR, worlds and objects are existing because satisfying its ontological (causal) rules and axioms. And not like in R. Hayaki's "free logic", who first formalized an ontology of *fictional* "world-stories", as a procedure for defining according to rules "non- existing" objects as referents of "dummy names", in her ontology of fictional stories in literature[13].

[13] Effectively, the conditions we are here attributing to our composite causal relation *R** are the same that R. Hayaki attributed to the stipulation relation *S* on non-actual objects as referents of dummy names in her free-logic (Hayaki, 2003). This depends, we repeat, on the fact that because of our foundation axioms we can be *possibilist* without being *conceptualist*.

**OGP:** For justifying the construction of world-stories, the binary accessibility relation $R^*$ (effectively, the composite accessibility relation $R_{\Gamma\circ\gamma}$) must satisfy the following conditions:

1. $R^*$ forms a *tree*, that is:

a. $R^*$ *is generated*. It has an origin point $<\gamma_n>$, for $n > 0$, in one of the actual worlds at the state $W_n$ of the universe evolution. $<\gamma_n>$, is thus $R^*$-related to other worlds at the successive level depth $n$+1. The original point is at level 0. The immediately stipulated worlds from level 0 are at level 1, from the level 1 are at level 2, and so on. We define thus as *secondary generators* $<\gamma_n>$, with the index $n \geq 0$ denoting the world-story level, all the worlds ("ancestors"), progressively originating a world-story of new levels of stipulated worlds/objects in a nested way,.

b. $R^*$ *is antisymmetric*. For no two distinct elements at two different levels, $R^*$ holds in both directions. Nevertheless, the symmetric relation holds between each pair of worlds of the same level that have the same origin, i.e., the same generator $<\gamma_n>$, worlds having a direct common ancestor. This confirms that the modal axiom **5** holds for such a logic[14].

c. $R^*$ *is anticonvergent.* I.e., the branches, once separated do not rejoin.

2. $R^*$ *is irreflexive***.** For granting this other important property of $R^*$, we have to recall that generally the reflexive relation is linked in ML to the axiom **T.** On the contrary, the (causal) accessibility relation $R^*$ de-

[14] Effectively, Hayaki says that in such a way the axiom **B** ($\alpha\rightarrow\Box\Diamond\alpha$), and not **5** ($\Diamond\alpha\rightarrow\Box\Diamond\alpha$), holds, because of the actualist, non pssibilist character of her ontology. Without such an actualist posit, **B** (the so-called "Browerian axiom") and **5** (or **E,** the so-called "Euclidean axiom") are effectively equivalent, and generally so are considered in many modal logic handbooks.

pends on the axiom **D** granting, by seriality, that the chain of the derivations is always closed. This is a new confirmation that *R** within our modal system **KD45** perfectly fits with the logic of the *R** relation.

3. *R* is intransitive***.** Such a condition grants that any *R**-generated world has a direct ancestor, a condition that a transitive relation cannot grant in principle. On the contrary, the "weak" transitivity of the Euclidean relation perfectly satisfies such a condition, confirming our interpretation. Newly, *R** in the iterated modal system **KD45** perfectly fits also with this third condition.
4. Between every pair of unraveled worlds/objects at the level *n*, each of them having a binary relation *R** with a common ancestor $<\gamma_{n-1}>$, it is possible to justify an anti-symmetric relation. This, in the symmetric case, allows to justify a *secondary* transitive-symmetrical-reflexive (=equivalence) between each pair of worlds/objects satisfying these conditions. The totality of these pairs constitute thus a new *domain* of predication – a new *natural kind* of actually existing things –, at the state *n* of the whole universe story-sequence. This confirms that each stipulation procedure of nested world- stories has a nested **KD45** structure.

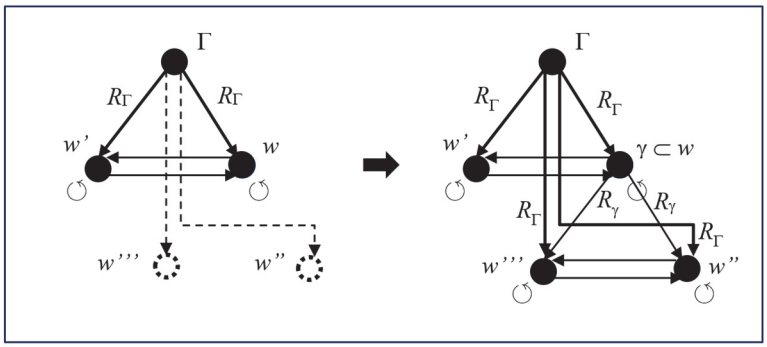



Figure 1. Scheme of **OGP** in the NR ontology. (Left) The worlds *w*" and *w*''' are potentially existing in the active power of the primary generator <Γ>. (Right) After being "unravelled" by the secondary generator <*γ*> within the world *w,* the worlds

*w''* and *w'''* become actually existing with all their elements. It is evident the iterated, nested **KD45** structure of the system.

Finally, the objectual QML of the NR formal ontology is a modified version, **Q1R*,** of the **Q1R** system. Indeed, because of our **OAE1-2** we can use a first-order stratified **Q1R*** system as the QML of NR, without supposing a free logic. Namely, all names, either proper (for individuals) or common (for natural kinds), have their referents, which are *possibly* existing, because of **OAE1**, or *actually* existing, because of **OAE1-2**.

**Q1R-model:** Generally, it is an ordered quintuple $<W,R,D,Q,V>$, where:

- $W$ is a set of worlds, effectively a sub-set of the universe {**W**};
- $R$ is the binary accessibility relation;
- $D$ is the domain;
- $Q$ is a function assigning to each world $w \in W$ a subset $D(w) \subseteq D$, i.e. the domain of quantification of $w$;
- $V$ is a valuation assigning for each world $w \in W$:
  - an object in $D$ to each term $t$, variables included;
  - a set of ordered $n$-tuples of elements of $D$ to each $n$-ary predicate, and
  - the set $D(w)$ to the existence predicate $E$.
- $V$ must satisfy the rigidity condition on terms **VRT**:

  **VRT:** $V(t, w) = V(t, w')$ for all $w, w'$ in $W$

**Q1R*-model:** for applying to NR the **Q1R** logic, the model is not the quintuple $<W,R,D,Q,V>$, as above, but a sextuple $<W, \gamma_n, R^*,D,Q,V>$, where the difference is given by:

1. $\gamma_n$ is an element "admitted" in $W$; and
2. $R^*$ is our **OGP**, on $W$; with $\gamma_n$ being its origin point.

Finally, in NR, a **Q1R*-model** must satisfy the rigidity condition **(VRT)** and the arboreal **SB** condition so re-defined:

**SB*:** If a non-actual object *z* appears for the first time in a world *w* (i.e., is not present in any ancestors of *w*, including $\gamma$), then *z* can appear only in *w* or its descendants.

## *2.6 Applications of NR to a Formal Ontology of the Evolutionary Cosmology*

### 2.6.1 The ontology of time and of complexity in the NR formal ontology

The *ontology of time* in NR supposes clearly an *Aristotelian* tense-logic and not a *Diodorean* one. Indeed, this latter is very poor and, overall, no ontology of *history* can be founded one the Diodorean ML semantics of time, where the "possible in time", i.e. $<\Diamond^t>$, is limited to the "present" ($n$), and the "future" ($\mathcal{F} > n$) , but not to the "past" ($\mathcal{P} < n$).

On the contrary, the notion of "history" suppose an *active power* of the past as to the present, just as the present has an *active power* as to the future, and not vice versa. In other terms, we need the so-called *Aristotelian theory of time* whose ML is, of course $\mathbf{S_t5}$. I.e., it is sound as to **S5** (Cocchiarella, 2007, p. 45), and not only as to **S4,** as it is for the temporal ML, $\mathbf{KT_t4.3}$, usually associated to unraveling (Blackburn, De Rijke, & Venema, 2002, p. 219).

Effectively the $\mathbf{S_t5}$ in NR is a nested $\mathbf{KD_t45}$, structure, compatible also with a multi-verse cosmological hypothesis, but where, differently from Aristotle, there is no necessity of supposing an infinity of time as to the past *within each universe*. Indeed, because of $\mathbf{OAE_{1\text{-}2}}$, there exists, within each universe, a time $n = 0$, cosmologically its initial "big-bang" among the potentially infinitely many in the QV – in NR, $\overset{\subset}{\mathbf{V}}$, because of $\mathbf{OAE_1}$,–, corresponding to the level 0 of the secondary generators from which all the world-stories of that universe start. In this way, we can define a Principle of Time Ontology (**PTO**), where the tense necessary operator "what is always the case", $<\Box^t\varphi>$, can be justified as follows:

$$\textbf{PTO}: \quad \begin{array}{l} \Diamond^t_k \varphi := \left(\mathcal{P}\varphi \vee \varphi \vee \mathcal{F}\varphi\right) \\ \Box^t_n \varphi := \left(\neg\mathcal{P}\neg\varphi \vee \varphi \vee \neg\mathcal{F}\neg\varphi\right) \\ \therefore \quad \Box^t \varphi \leftrightarrow \left(\neg\Diamond^t\neg\varphi\right) \end{array} \qquad \text{where:} \begin{cases} \mathcal{P}\varphi & \text{for } k < n \\ \varphi & \text{for } k = n \\ \mathcal{F}\varphi & \text{for } k > n \end{cases}$$

Of course, an "arrow of time" is modally defined, because there exists an ontological difference between the two temporal possibility operators, $<\mathcal{P}, \mathcal{F}>$ in NR. Indeed, the "past" is made of things already "unraveled" at the levels $<n\text{-}k>$ of the universe becoming, when they actually *existed* as "individuals". Afterward, they continue to exist in their world-story no-longer as actual individuals, but *virtually* as *causally active* "parts" in the new "wholes" actually existing, as *more complex things,* at the new actual level *n* of their world-story. On the contrary, the "future" is made of things not "yet" unraveled, and hence not yet determined in their proper nature, so that they cannot exert any "backward" causality.

**Remark:** In other terms, the direction of "the arrow of time" in the time ontology of NR coincides with the direction of the arrow of the causal implication, of the "causal entailment", i.e., "the cause precedes always its effect" ($\rightarrow_c$). It precedes "ontologically" and hence also "temporally" the effect. In this sense, the fact that behind the direction of the *ontological* "causal entailment" there is the opposite direction of the *logical* and hence *epistemological* "strict converse implication" ($\twoheadleftarrow$), suggests a *definitive clarification* of the ambiguous notions of "backward" (from the "effect" as to the "cause") and/or of "downward" causation (from the "whole" as to its "parts"), often associated to the notions of "emergence" and/or of "complexity" in natural sciences (De Haan, 2006; Mazzocchi, 2008). Only if we suppose the "atomism" in physics, and the "logical atomism" in logic and ontology, we confuse the ontology of causation with its logic and epistemology. "Backward" and "downward" are the verses of the *inferential* process between propositions, not of the *ontological,* causal process that is the *referent* of the propositional inference (see Quine's precious

criticism to Russell's and Lewis' confusions on this regard: §2.11, and Aquinas' suggestion of the inversion between the logical and the ontological verses of the implication: §2.1.2). The emerging complex system, as far as it is related with the QFT notions of "phase coherence domain" and hence with its "duality principle" ("force field/its quanta" of QFT, *vs.* "statistical wave function/particles" of QM: see §1.2) supposes that the "parts" (e.g., the electrons in an atom) are no longer actually existing "individuals", with whom it is possible to "interact" as causes and/or as effects. The new individual *actually* existing is the "whole", e.g., the phase coherence domain of an atom, in which the electrons exist *virtually,* i.e., as quanta of the electronic force fields. The dynamic "whole" of the atom does not "interact" with its dynamic "parts", the quanta of the electronic field, but determine their quantitative properties, i.e., the characteristic quantum numbers of the electronic field. In a word, the parts are no longer actually existing individuals as "free" electrons or protons, they *were* individuals, e.g., in the high temperature plasma of the sun, before the "capture" of their force fields in the new phase coherence domain of an atom. When the new, more complex, individual comes to the *actual* existence, it is because its components passed to the *virtual* existence in it, and *vice versa.* When the "parts" were actually existing individuals, the "whole" existed only *virtually* in the causal power of its future parts. *Virtus* in Latin means literally *force*!

### 2.6.2 The ontology of natural kinds in NR

From all the preceding relations, we can define a formal ontology of natural kinds in NR. It is based on the notion of nesting of the physical causality, and so it is deeply different from the CNR ontology of natural kinds (Cocchiarella, 2007), where the causal explanation of natural kinds and of their hierarchy is purely *hypothetical.* Evidently confusing the natural and mathematical science inferential method, on one side, and the inferential method of formal semantics and formal ontology, on the other side. The main elements of NR formal ontology of the natural kinds can be synthesized in the following principles:

- A principle of Stratification for Natural Kinds **(SNK)** holds in NR, i.e.:

$$\mathbf{SNK}: \left(\forall^{k}_{n}A\right)\left(\forall yA\right)\Box^{C}_{n}\left(E!(a)\rightarrow\left((a=y)\wedge\left((a\wedge y)\ {}_{k}\ni\mathbf{A}\right)\right)\right)$$

  Where $A$ is a natural kind (species) predicate ranging over a domain of physical things (e.g., quarks, or, at higher *n,* protons, neutrons, or at even higher *n,* different species of atoms, etc.), and the apex $k$ to the quantifier signifies that the quantifier is ranging over natural kind predicate variables, all causally constituted at some level $n$ of the universe evolution. The same $k$, appears as an index to the co-membership predicate $\ni$, with the meaning that a give domain of individuals are "admitted" within their own natural kind, i.e., they constitute a *species* of individuals as a domain of real individual things sharing the same ancestor $\gamma_{n\text{-}1}$.
- A principle of Stratification for Natural Properties **(SNP),** shared by things admitted, either by the same, or by different natural kinds (e.g., the electromagnetic charge shared by quarks, protons, electrons…), holds in NR, i.e.:

$$\mathbf{SNP}: \left(\forall^{e}_{n}F^{j}\right)\Diamond^{C}_{n}\left(\exists^{e}_{n}x_1,...,x_j\right)F\left(x_1,\ldots x_j\right)$$

  Where we recall that the index $e$ signifies that the quantifiers are ranging over actually existing natural property predicates $F$ and individual variables $x$.
- A Principle of Stratified Rigidity (**PSR**) holds in NR. Such a condition occurs for a predicate $F,$ at the level of the universe evolution $n$ in which it is generated within a given world-story, and holds for all the successive $n+m$ levels (with $m \geq 0$) of the universe evolution, i.e.:

$$\mathbf{PSR}: \Box^{C}_{n}\left(\forall_{n}F\right)\left(\exists_{n+m}G\right)\left(Rig(G)\wedge\left(\forall x_1,...,x_j\right)\left(F\left(x_1\ldots x_j\right)\leftrightarrow G\left(x_1\ldots x_j\right)\right)\right)$$

- From all the precedent definitions, a Principle of concrete Existence **(PCE)** for physical individuals (= things) derives, defined as follows:

$$\mathbf{PCE}: \left(\forall^{k}A\right)\left(\forall yA\right)\Box^{C}_{n}\left(E!(a)\rightarrow\left(y=a\right)\right)$$

From this it emerges that any physical being exists as an individual, *a,* only as a member *y* of a natural kind (species), and hence as the outcome of a world-story, shared by other individuals of the same species, inside the history of the universe.

- Finally, a Principle of Biological Individualization **(PBI)**, different from **(PCE)**, holds in NR for the concrete exist- ence of biological individuals as formalizing the *epigenet- ic,* self-organizing, factors (i.e., chemical signals over its genetic constraints), by which the organism ontogenesis "individualizes" for itself (e.g., by activating/de-activating some sequences of DNA) the *genetic* factors of its specific DNA that could be shared by a homozygote twin, i.e.:

$$\mathbf{PBI}: \left(\forall^{k_l} A\right)\left(\forall yA\right)\Box_n^C\left(E!(a)\rightarrow\left(y \rightleftarrows a\right)\right)$$

Where the index *l* affixed to the apex *k* of the natural kind quantifier means that this is a natural kind of living individuals, and $\leftrightarrows$ designates the (coalgebraic) dynamic notion of "equivalence by bi-similarity" (or "bisimulation") (Sangiorgi & Rutten, 2012), in the sense that each modification in *y* is matched in *a,* and vice versa.

### 2.6.3 The ontology of the conceptual realism in the NR formal ontology

Finally, for justifying that such an ontology is a "natural *conceptual* realism", NCR, and not only a "natural" realism, NR, we have to justify the passage from the natural realm of things, to the *mental realm of concepts*, because the NR calculus is not justified, like in Cocchiarella's CNR, by a conceptualist axiom like the Fregean "comprehension axiom" on which the calculus of CNR ultimately depends.

Indeed, in the case of the most complex among the living beings, i.e., the animals and finally the humans, it has to exist the faculty of performing *cognitive simulations* of the outer world. This representational faculty can be descriptively sketched as the faculty of *mirroring* the *ontic* co-membership, ϶, genus-species (or species-individuals) in nature, into the "reversed" *logic* membership, $\in$, subset-superset in mind. Formally, this can be justified in the framework of the Category Theory logic, where it is possible to demo-

nstrate the *dual equivalence* ($\leftrightharpoons$) between the categories of the *modal coalgebras,* defined on NWF sets, and of the *modal Boolean algebras* for a "contravariant" application of the same functor Ω, inverting all the "arrows" (morphisms) and the "composition orders" between them. I.e., $\mathbf{CoalgΩ} \leftrightharpoons \mathbf{BAlgΩ^{op}}$ (Venema 2007; Basti 2015). This means that the inductive formulas of the modal predicate calculus (algebra) are dually satisfied by the corresponding co-inductive formulas (coalgebra), according to the following scheme: $\Box_n\left([\alpha \in \beta] \xleftarrow[\Omega]{\rightleftarrows} [\![\alpha \ni \beta]\!]\right)$, where $\xleftarrow[\Omega]{\rightleftarrows}$ means a dual equivalence, functorially induced from the coalgebraic (dynamic) structure into the algebraic (logical) one, determining a modal *local truth*. Therefore,

- A Principle of Cognitive Induction **(PCI)** holds in NCR:

$$\mathbf{PCI}: \left(\forall^m F^j\right)\left(\forall^m x, a\right)\Box\Diamond^C_{n+1}\left(\exists! a\left(\left(x \rightleftarrows a\right) \wedge \left(F\left(x_1,...,x_{j-1},a\right)\right)\right)\right)$$

Where, the index *m* (mental) emphasizes that we passed from the *natural* to the *conceptual* realm, at the (*n*+1) *abstract* level as to the *n* level of the actual existence. Consequently, *x* is a mental variable denoting a generic individual, *a* is a mental constant denoting a given existing individual, and the double modal operator, $\Box\Diamond^C_{n+1}$, emphasizes that we are speaking about a "causally necessary possibility", that is, a *natural faculty* of the cognitive agent as such.

Finally, in the case of the singular denotation, it is proper of the human mind to re-define inductively onto a singular individual, also the predicate and not only its argument. This means that we can define in NCR:

- A Principle of Generalizing Abstraction, **PGA**:

$$\mathbf{PGA}: \quad \Box_n\left[\left(a \ni A\right) \xrightarrow[\Omega]{\rightleftarrows} \left(\left(a^m \in A^m\right) \rightleftarrows \varphi\alpha\right)\right] \wedge \left(\left(\varphi\alpha \Rightarrow \forall^m x\left(\varphi x\right)\right) \equiv \mathbf{A}\right)$$

Where, *a* is an individual thing, *A* is its natural kind, $a^m$, $A^m$ are the correspondent mental objects, $\varphi\alpha$ denotes meta-logically the bisimilar "definite description" of *a*, characterized by the identity between the predicate and its argument in denoting individuals. The right term of the conjunction is thus a formulation of the "principle of universal generalization" of the classical predicate calculus, valid only for mental objects, and <**A**> is a symbol denoting an abstract class, corre-

sponding to the extension of the predicate <$\varphi$>.

**Remark:** It is evident, from the left term of such a conjunction, that in such a way it is possible to give a formal original solution of the problem of reference, in which a definite description, $\varphi\alpha$, is functorially induced, $\leftarrow\!\!\!\!\underset{\Omega}{\rightleftarrows}$ by its referential individual object *a*, through the "equivalence by bisimilarity", $\leftrightarrows$, with the correspondent mental objects. That is, where every modification in one of the equivalence terms, induced by the real referent, is "balanced" by a correspondent opposed modification in the other one, so to make invariant the relation. This is the NCR formulation of the notion of "truth as *adaequatio*". This is, moreover, the ontological counterpart of the neurophysiological evidence of the "mirroring" within one only phase coherence domain of a brain state modification (algebra) and the "contravariant" thermal bath state modification (coalgebra), in the QFT interpretation of brain dynamics as "open system" in fundamental physics (see above §1.3 and (Freeman & Vitiello, 2006; Capolupo, Freeman, & Vitiello, 2013; Basti, 2013b, 2015, 2017).

### *2.7 The NR logic is a paraconsistent logic*

It is well known that there exist significant elements of paraconsistent logic both in Ancient and Middle-Age logic (Gomes & D'Ottaviano, 2013). From this historical point of view, we can now add also Aquinas to such a list.

Anyway, as we saw in §2.2, the notion of "*logical* entailment" – i.e., "*p* entails *q*", that is, "*q* follows logically from *p*" – as the proper semantics of Lewis' strict implication, <$p \twoheadrightarrow q$>, is ultimately a very strong way – effectively the strongest – for affirming the logical truth of the so-called "principle of Pseudo-Scotus". That is, the "principle of explosion", **PE** (*ex contradictione sequitur quodlibet*), expressed in the modal formula: <$(p \wedge \neg p) \twoheadrightarrow q$>, or <$\Box (p \wedge \neg p) \rightarrow q$>, that is, "*necessarily* from a contradiction anything derives". As we already recalled in §2.1, according to (Huges & Cresswell, 1996, p. 203) a list of paradoxes is deriving from it and by the notion of logical entailment in Lewis' formulation.

Hence, following Lewis himself, we saw that if we want to avoid **PE** and the other related paradoxes, we have to exclude before all the so-called "principle of the disjunctive syllogism": $\left(\left(p \vee q\right) \wedge \neg p\right) \rightarrow q$ .

Namely, we have to refer to the so-called *relevance logics* (Huges & Cresswell, 1996, p. 205), i.e., it is necessary to define a valid criterion of *relevance* of a premise as to a given conclusion, and hence to refer to the notion of *paraconsistent negation* (Béziau, 2000; Béziau, 2005).

It is evident that, as we have already anticipated, the ML of NR avoids all the paradoxes related to the notion of the "logical entailment". Before all, the truth-table of the converse implication, $<p \leftarrow q>$, states that from the false only the false can be inferred. So, the modalization of the converse implication, – i.e., $\left(p \rightarrow_C q\right)$, or, $<\neg\Diamond(q \wedge \neg p)>$ –, implies that the semantics of the "*causal* entailment"– i.e., "*q* entails *p*", that is, "*p* precedes causally *q*" – makes false the **PE**, that is $<\neg[(p \wedge \neg p) \rightarrow_c q]>$. In other terms, neither the "principle of the pseudo-Scotus" nor the "principle of the disjunctive syllogism" – i.e., $\neg\left(\left(p \vee q\right) \wedge \neg p\right) \rightarrow q$ – hold in the NR logic.

On the other hand, it is evident that a logic of causality based on the "converse implication" is naturally a *relevance* logic, since only a true premise (denoting the cause) can imply a true conclusion (denoting the effect).

It is evident too, moreover, that the principle of "iterated modality" and the consequent "stratified" nature of the necessity operator in the NR coalgebraic QML on NWF sets in which unbounded chains of set inclusions are allowed, opens NR logic to the possibility of an original interpretation in it of the *paraconsistent* negation. This true, as far as both are based, not only on the refusal of **PE** and of its trivial consequence, but also on the principle of the non-coextensive character of an affirmation with its negation in contradictory statements (Béziau, 2000). The stratified, nested character of the necessity operator in NR logic opens thus the possibility for an original version of the *constructive* use of the contradiction, typical of the paraconsistent logics. Indeed, because of the nested character of the causal necessity operator, while the negation $<\neg p>$, contradicting

the affirmative <*p*>, negates <*p*> at its proper necessity level, the affirmation <*p*> potentially includes all the other propositions not yet unraveled by the iterated modality procedure illustrated in §2.3. From the ontological standpoint, this ultimately depends on the axioms **OAF**$_{1\text{-}2}$, from which the action of the primary generator <Γ> emerges as including all the levels and all the branches of the iterated modality hierarchy – both the levels already unraveled, and the levels not yet unraveled – in a *causal,* not *logical* way. That is, in which the contradiction does not propagate itself to the lower levels of the argumentation tree, given that no *set total ordering* is allowed in NWF coalgebraic logic.

In other terms, the information (truth) in the causal implication is not conserved between the antecedent(s) and *all* the consequent(s). In fact, at each level of the unraveling procedure the *actual* information *increases*, since a new structure emerges, as absolutely, logically, *unpredictable* from the precedent ones. Such an emergence however, is at the cost of a decrease of the *potential* information included in the precedent levels, since at each of them the procedure chose one only of the two possible branches <1, 0> that were available.

Of course, much more analysis is needed, for deepening the relationship between the **KD45** modal logic, and its iterated, nested structure in our NR formal ontology, and the paraconsistent negation, by expanding the analysis between modal logics – particularly, the "un-named vertex <¬,>" of the modal square of oppositions – and the paraconsistent negation. Such an analysis has already been developed, for instance in (Béziau, 2005), but limited to **S5** and **S4** systems and to a four-valued modal system such as **M4**. On these topics, much more intriguing, and hence requiring a much deeper examination, are the most recent (Béziau, 2012; 2013), extending the analysis to the more powerful hexagon, and the modal hexagon, of oppositions, used also at the meta-logical level.

Moreover, this initial survey of the relationships between NR logic and paraconsistent logics, displays a possible connection with the hierarchy of logical systems, and of the related algebras, proper of Da Costa's paraconsistent logic, $C_\omega$ (Da Costa & Alves, 1977). Both

approaches indeed are developed on the basis of a positive intuitionistic logic, so that both admit for systems of level $n$ (where $1 \leq n < \omega$), that consistent formulas of the type $< p^n \wedge \neg p^{n+1} >$ hold in a theory. In addition, we can say that the core of NR formal ontology is precisely for supporting such a possibility in a *constructive* way, not only from the logical, but overall from the *ontological* standpoint, so to finish definitely the cloying, ideological debate between the supporters of classical metaphysics and the supporters of the evolutionary theories in natural sciences.

Finally, a consideration about the relationship with dialectical logics and more generally with *dialetheism*. Both the logic of NR and many paraconsistent logics, the $C_\omega$ included, validate the law of non-contradiction, i.e., $< \Box \neg(\alpha \wedge \neg\alpha) >$, even though both invalidate **EP**. In this sense, both of them are deeply different from the "dialetheism", i.e., the metaphysical position, typical, for instance, of the Hegelian *Science of Logic*, according to which there exist logically *true* (ontologically *real*) contradictions. Hence, both NR and paraconsistent logics can give a contribution to the overcoming of an ideological approach to this problem, characterizing the philosophy of logic during the XIX, and a large part of the XX centuries (Da Costa, Béziau, & Bueno, 1995, pp. 112-113).

## 3. General conclusions and further perspectives

In this paper we offered for the first time a semi-formal general presentation of the *Natural Realism* (NR) formal ontology, according to the double perspective of:

1. Providing a formal ontology for the paradigm shift actually involving the fundamental physics, based on QFT, as far as it is irreducible to QM, and it is related to an evolutionary approach to cosmology. Such an evolutionary vision involves also the same foundations of the mathematical laws of physics. In other terms, "physics is legislated by cosmogony", according to the visionary expression of J. Archibald Wheeler.

2. Providing a formal ontology for the *natural* realism endowed with a *suitable* logical calculus, able to justify the construction of arboreal structures of quantification domains of growing complexity, based on a coalgebraic QML on NWF sets. Such a logical calculus is a possibilist version of Hayaki's *nested stipulation principle,* because based on a modal version of the logic of *the converse implication*. Namely, the logic of the Aristotelian *formal causality*, as dynamically generated by an acting causality on the matter indeterminacy, determining the *emergence* (*eductio* in Latin as opposed to *deductio*) of a new form in it. That is, the logic of the dynamical emergence of collective behaviors from the quantum vacuum, like as many "phase coherence domains" of fields, according to the QFT formalism of the categorical duality coalgebras-algebras – effectively, between *q*-deformed Hopf coalgebras and algebras. In fact, this formalism was originally developed by its authors independently from any knowledge of Category Theory logic, for justifying mathematically the energy balance – with the inversion of all the energy arrows – between the thermal bath (coalgebra) and the system (algebra) in a "doubled" Hilbert space, for modeling open quantum systems in a *far-from-equilibrium stability* (Blasone, Jizba, & Vitiello, 2011). This modeling is indeed in principle impossible in quantum statistical mechanics, where, because an ultimate equilibrium in the infinite volume has to be anyway granted, the operator algebras can admit the usage of only *covariant* Von Neumann algebras for representing quantum interacting systems, in a *near-to-equilibrium stability* (Hollands & Longo, 2016).

In the first section of this work, after a sketchy presentation of the QFT approach to fundamental physics, from cosmology, to the standard model of the elementary particles, to the physics of the condensed matter, living matter and neuropils included, we presented in the second section a summary of the NR formal ontology. Its principal merit, according to us, is that it is able to satisfy, from the ontological side, the requirement for an ontology of the natural realism, able to cope with the "paradigm shift" that quantum physics and, more specifically, QFT, as irreducible to QM and to quantum

statistical mechanics, is imposing to the modern vision of fundamental physics and cosmology.

The semi-formal presentation of NR formal ontology, object of the whole second section of this work, is based on Aquinas' suggestion that the logic of the causal necessity for the *emergence* (in Latin, *eductio,* "eduction") of new natural forms of matter organization from the potentiality of the Aristotelian "primary dynamism" (*próte dynamis*) of the "first matter", is the (modal) logic of the *converse implication.*

Finally, there exists a straightforward evidence that the logic of NR is ultimately a paraconsistent logic, because it satisfies the **NC** principle but not **EP**. Because of $\mathbf{OAF_{1\text{-}2}}$ and the hierarchical nested semantics it generates, indeed, it is evident that a paraconsistent negation principle holds in NR logic, because of the non-coextensive character of affirmation and negation in it, and the consequent stratified character of the rigidity principle.

Such a relationship with a paraconsistent logic and with a hierarchical paraconsistent logic as Da Costa's $C_n$ logic (Da Costa, Krause, & Bueno, 2007) , and, more generally, the relationship between NR formal ontology with its causal reference theory, and with Da Costa's notion of *pragmatic truth* (Da Costa, Bueno, & French, 1998) requires a specific further inquiry.

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