INNOPOLIS UNIVERSITY

Classes 14-15

Formal Philosophy. The Modern Age:

Language & Cognitivism

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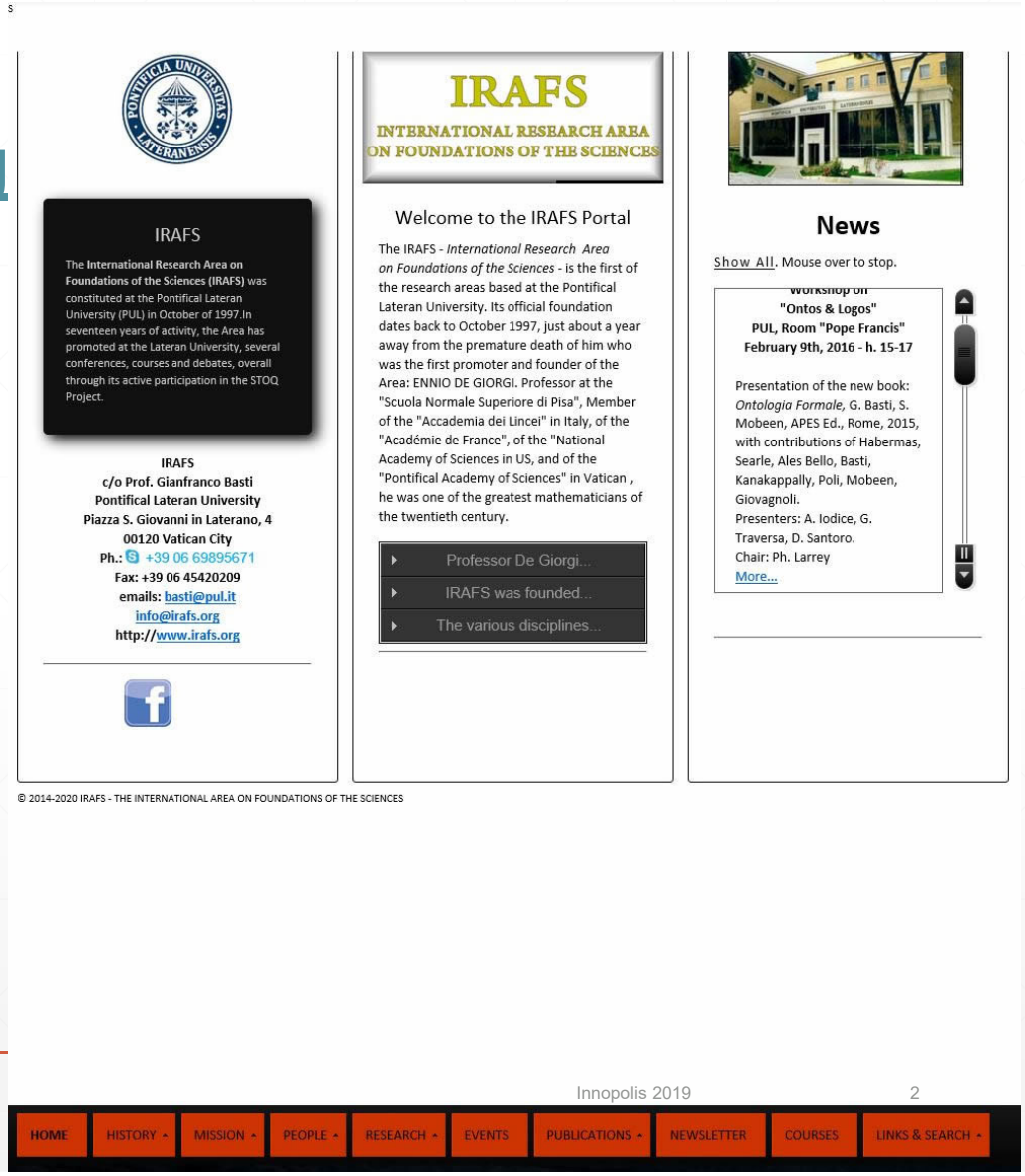
Course: Language & Perception

Syllabus I Part (1-2/11/2019)

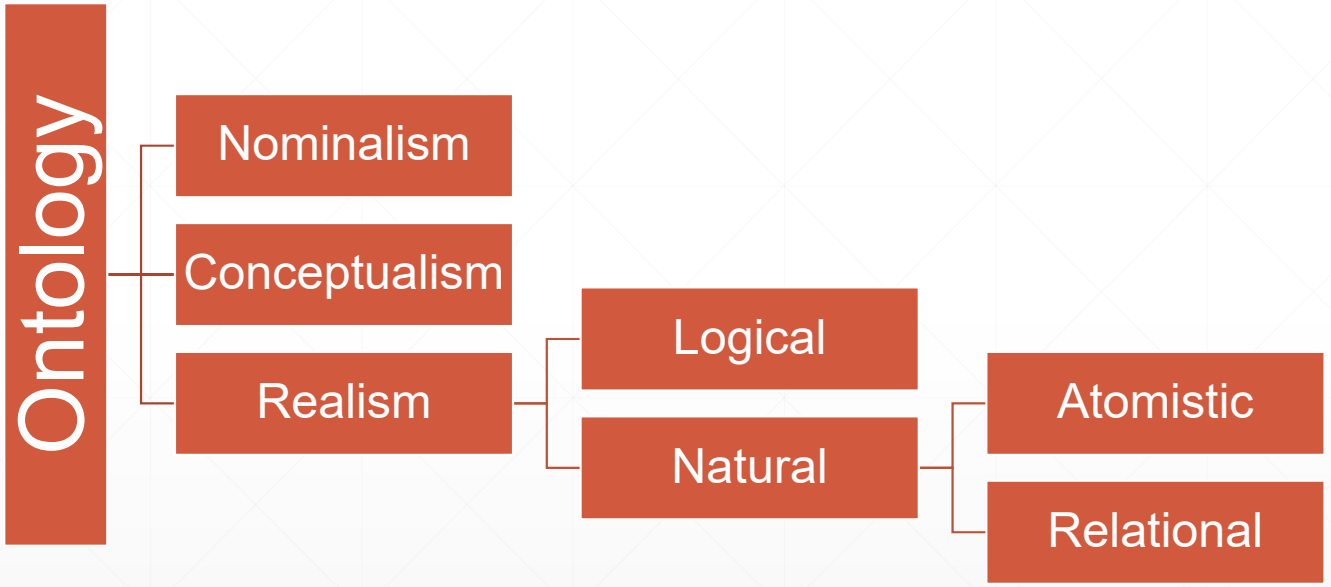
Syllabus II Part (8-9/11/2019)



Summary

- The Galilei's affair: apodictic *vs.* hypothetical method in modern Galilean science
- Descartes' first development of analytic (algebraic) geometry and the supposed apodictic value of mathematical sciences
- This is based on self-consciousness as cognitivist foundation of self-identity of a logical tautology, extended by Newton to the self-evident character of the three laws of Newtonian mechanics (*hypotheses non fingo*).
- This is made explicit by Leibniz's distinction between analytic and synthetic judgements, as well as – following Newton – by its empiricist counterpart by Hume, Locke and Berkeley, and finally, by the Kantian theory of the synthetic a-apriori judgements about pure mathematics and physics.
- Refs.: 3. (ch. 1), and 10.

Formal Ontologies Scheme



The Gailei affair and the birth of modern science

The Galilei Affair: hypothetical *vs.* apodictic method in modern science

Hypothetical *versus* apodictic argumentation I

- Distinction in logic between **validity** and **soundness**
  - **Validity:** An argument is valid iff the truth of its premises entails the truth of its conclusions, at every step of the logical argumentation procedure. The corresponding conditional of a valid argument is a **logical truth** (i.e., necessarily true or true in any interpretation or «possible world», namely, in the case of propositional logic, it is a **tautology**), and its negation is a contradiction. In this way, the conclusion is a **logical consequence** of its premises.
    - Validity does not imply **soundness,** i.e., an argument can be valid even though the premises are not necessarily true (validity depends only on the **logical form** of the argumentation).
  - **Soundness:** An argument is sound iff:
    1. The argument is **valid**
    2. All of its premises are **true**
  - **Soundness** and **strong soundness** of deductive systems
    1. A deductive system *S* is **sound** iff any sentence *P* provable in it, is also true in all its interpretations within a given language *L:* **if** $\vdash_S P$ **then** $\models_L P$.
    2. A deductive system is **strongly sound (apodictic)** iff any *P* derivable from a set Γ of premises is also a logical consequence of Γ (i.e., any model making all the elements of Γ true, makes true also *P*): **if** $\Gamma \vdash_S P$ **then** $\Gamma \models_L P$.

Hypothetical *versus* apodictic argumentation II

- **Apodictic (strong soundness)** character of any syllogistic argumentation in its **metaphysical use,** as far as its soundness supposes that the necessary truth of premises is an **ontological truth** (the predicative sentences refer to some essential properties of the beings concerned) and not a simple **logical truth.**
- E.g., the classical *in Barbara* form:

All humans are mortal	M&P
All Greeks are humans	S&M
All Greeks are mortal	S&P

- Whose correspondent conditional in predicate logic is:

$$\left(\left(\forall xMx \rightarrow Px\right) \wedge \left(\forall xSx \rightarrow Mx\right)\right) \rightarrow \left(\forall xSx \rightarrow Px\right)$$

Hypothetical *versus* apodictic argumentation III

- Hypothetical reasoning: **valid** even though its premises **are not necessarily true** (true in any interpretation of a given language, or in any model of the logical system).
- E.g., in the classical *modus ponens:*

  If the sun is shining, then it is light, but, the sun is shining: hence it is light

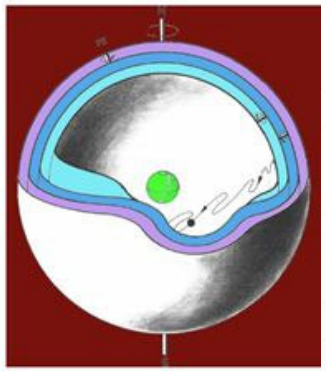
  The premises of such an argument are true (in the factual sense), and hence the argument is sound only during the day (from morning to afternoon included: **model 1**), but are false during the night (from evening to night included: **model 2**). Nevertheless the argument is always valid because both models are interpretations of the following tautology or logical law (= *modus ponens*), constituting its correspondent conditional:

$$\langle((p \rightarrow q) \wedge p) \rightarrow q\rangle$$

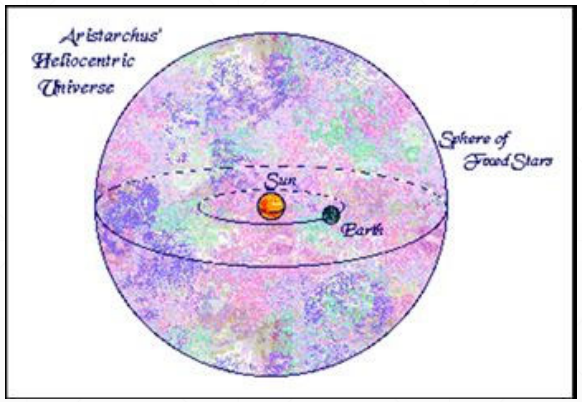
A false interpretation of hypothetical arguments

- **A false interpretation of Bellarmine's suggestion** of the hypotehtical character of the mathematical demonstrations of the Copernican and Galileian physics, and that Galilei had originally accepted, ignited the dispute that led Galieli to write the polemical book of the *Dialogue* and hence led him to his condemnation of 1633.
- Such a false interpretation **depends historically** on the solution that Geminus proposed in the 2nd century B.C. to reconcile the astronomical observations and measurements made by Aristarchus of Samos (310-230 B.C.), with the Aristotelian theory of the concentric heavenly spheres which had the Earth as the universe centre.
- A theory that Aristotle in his *Metaphysics* borrowed from the **purely mathematical, non-physical, theory of his contemporary Eudoxus** (408-355 B.C.), and which in the 2nd century A.D., will be taken up by Ptolemy (90-165 A.D.), adjusted with his famous theory of the «epicycles», in order to save phenomena.

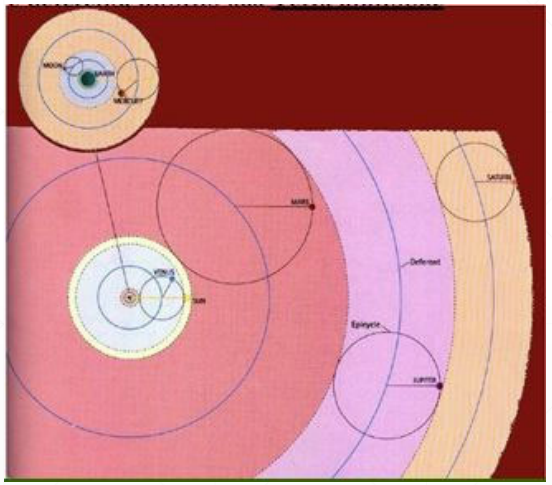
Eudoxus, Aristarchus and Ptolemy



Eudoxus (408-355 B.C.)



Aristarchus (310-230 B.C.)



Ptolemy (90-165 A.D.)

The Simplicius account

- According to the account of Simplicius of Cilicia (490-560 A.D.), commentator of the Aristotle's *Physics* and quoted by Stillman Drake in his book that reconstructs the Galilei question (Drake 1990, 59-60), the Geminus solution is the following:

Gemino's commentary, inspired by Aristotle's ideas, is the following (…). Astronomy explains only those things that it can establish by means of arithmetic and geometry. In many instances the astronomer on one hand and the physicist (i.e., the natural philosopher, in the Aristotelian sense of the word, *my note*) on the other, will attempt to prove the same point, for example that the Sun is very large and the Earth is round; but they will not proceed along the same path. **The physicist will demonstrate each fact with considerations about essence and substance, about forces**, about how good things are as they are, or about generation and change. **The astronomer will demonstrate things based on the properties of the figures or of the sizes or by way of the quantity of movement and time**, appropriate to it. **In many cases, a physicist can even reach the cause**, observing the creative force; but the astronomer, when he demonstrates facts from external conditions, is not qualified to judge about the cause, as when for example he affirms that the Earth or the stars are round. And perhaps he does not even want to ascertain the cause, as when he considers an eclipse, and other times **he invents, *by way of hypothesis* and affirms certain expedients through which *the phenomena will be saved*** (my italics).

Two main problems

1. **Non explanatory character** of the mathematical hypotheses of the «astronomer» because the explanation of phenomena can be only in **causal terms,** that is in terms of the «forces» depending on the «essences» of the bodies, objects of the «philosopher» inquiry.

2. **Fictional character** of the mathematical hypotheses of the «astronomer» ← confusion between **soundness** and **validity** of the hypothetical arguments. I.e., saying that they are not sound for all models, but only for some of them, does not imply that they are unsound at all, and hence «fictional». The universal validity of a hypothetical argument does not depend on its universal soundness!

- → **Questionable** reaction of Galilei:

1. Who vindicated the character of **absolute certainty** of the heliocentric mathematical hypothesis *vs.* the ptolemaic one, after the empirical confirmations of the former by his telescope.

2. Who vindicated the **ontological and hence the apodictic value** of the mathematical explantions of astronomy on theological, neo-platonic basis. That is, the mathematical laws of nature are true because direct expressions of God's thought → **ontotheological limit** of his metaphysics.

Essentialism, phenomenalism, and the Enlightenment scientism

- False interpretation of the mathematical hypotheses as «fictional» → **refusal of the hypothetical character** of the method of modern science during the XVI-XVIII centuries, till the discovery of the non-Euclidean geometries, with the consequent crisis of foundations in mathematics, and the birth of the «new physics», during the second half of XIX cent.
- Two ways for justifying the **apodicitic non-hypothetical character** of modern science argumentations at the beginning of the Modern Age:

1. **Essentialism:** the mathematical knowledge of nature is the only way for attaining at **the true essence of nature**, according to a renewed approach to the Pythagorean and Platonic metaphysics of nature (Galilei, Descartes, Leibniz, Spinoza).
2. **phenomenalism:** according to the famous Newtonian *hypotheses non fingo*, → apodictic nature of mathematical demonstrations of modern science would not be based on the presumed capacity of the mind to intuit the mathematical essence of physical reality beyond the phenomena, but rather on the Cartesian **principle of evidence** in Newton and Kant epistemological interpretation and generally in all modern science after them, till Riemann.

- Both interpratations anyway are aimed at granting the non-hypothetical character of the scientific demonstrations → interpretation of the modern science as the **true metaphysics of nature** → Enlightenment **scientism.**

The Essentialist position: A Koyré quotation from his *Introduction to Plato*

- If you claim a superior state for mathematics, and what is more, if you attribute to it a real value and a dominant position in physics, then you are a Platonist. If, instead you see in mathematics an abstract science that has less value than physics and metaphysics which deal with real beings, if you affirm that physics does not need any other base than experience and it should be constructed directly on perception, and that mathematics should be content with a secondary and subsidiary role, then you are an Aristotelian. In this debate, one places doubt not on the certainty of geometrical demonstrations, but on Being; and not even on the use of mathematics in physics – not even the Aristotelians would have denied the right to measure that which is measurable and count that which countable – but rather the structure of science and therefore the structure of being. (…) It is evident that for the disciples of Galileo, as for his contemporaries and predecessors, mathematics means Platonism. (…) The Dialogue and the Discourses thus tell us the story of the discovery or better the rediscovery of the language spoken by nature. They tell us how to question it, i.e., they contain the theory of that experimental re-search in which the formulation of postulates and the deduction* of their consequences precedes and guides observation. This later, at least for Galileo is a «factual» proof. The new science is for him an experimental proof of Platonism (Koyré 1980, 160. 163. 167).

Modern Platonism and the centrality of evidence

- The last quotation of Koyré expresses perhaps in the best way the core of **a modern interpretation of a Platonic ontology of truth** in logic and mathematics, shared by many modern mathematicians, Gödel included.
- At the same time, **the centrality of the question of "evidence"** as the foundation of truth – it does not matter whether it concerns the self-evidences of the Ideal essences as the ultimate intellectual objects of the abstract scientific praxis, or the self-evident *Erlebnisse* of the *Lebenswelt* with all their anthropological significance – emphasizes **the true motivations of the post-modern "linguistic turn" of the axiomatic method**, applied before to the mathematical logic, and henceforth to the philosophical logic, i.e., to the modal and intensional logics. **Let us sketch briefly the motivation of such a turn.**

The evidence at the core of Descarte's and Newton's epistemology

- **The *evidence*** – which is clearly a state of consciousness and therefore a property of a subject, either individually, collectively, or transcendentally intended – **is the core of the modern epistemology** since Descartes' *Discourse on the Method of Rightly Conducting One's Reason and Seeking Truth in Science* (1637) and Newton's *Treatise on Optics* (1704).
- Indeed, if, following an original suggestion of J. Maritain, we can synthesize **the turn from the Aristotelian to the Modern notion of science** as the passage from science as ***cognitio certa per causas***, "undoubted knowledge through causes", to science as ***cognitio certa per leges***, "undoubted knowledge through laws" (Basti, 2015), this turn has in the notion of evidence its key-point.
- Also in the Aristotelian sciences – the physical, the mathematical, and the metaphysical ones –, indeed, there are laws, but **these laws have a *causal* ultimate foundation in the "things",** and **in their "*real* (causal) relations", not in the mind thinking at them.** Even the abstract objects of pure logic and pure mathematics have in the *formal* abstraction from quantitative and qualitative properties of things the ultimate foundation of their truth.

Descartes' evidence

- On the contrary, the *law-like necessity* and therefore the *truth* of the rational, philosophical and scientific modern thought, from the First Rule of Descartes' *Discourse* on, **are founded on conscious "evidences**", concerning both rational objects and their rational relations, and not on the real things and their causal relations.
- The *First Rule*, indeed reads: **"Never accepting anything as true, if I didn't have *evident* knowledge of its truth"** (Descartes, 2007). Namely, it is "knowledge" that, either is "evident" because it is a theorem, or it is "self-evident", because it is an axiom of some deductive procedure.
- In other terms, the evidences here concerned **are not the "common sense" evidences of the Aristotelian physics and metaphysics** that led to clamorous mistakes such as the "geocentrism" in cosmology, **justifying the modern "methodic doubt"** about all those "old" philosophical certainties.

Descartes' and Galilei's evidence *versus* Aristotle's evidence

- This is the core of the "Galilean science" method, according to which **the mathematical laws of mechanics** – effectively, **the geometrical laws of kinematics** as the "geometrical science of motions" – **are *a priori* with respect to the empirical data**, and not *a posteriori,* because abstracted by real relations, like in the Aristotelian epistemology.
- This extension of Descartes' four *Rules* based on evidence, **from the original application to the only abstract realm of mathematics** and geometry; also **to the self-evidences of physics and metaphysics** is the core of the *Discourse* "new method".
- Given that – and we must always remember this fact –, the *Discourse* was firstly published as the **"Introduction" to an *Essays* volume** including not only Descartes' ***Treatise on Geometry*** – effectively, his fundamental pioneering work on an "algebraic geometry" –, but also his **Treatises on *Optics* and *Meteorology***. *Optics,* indeed, is among all the physical sciences, the physical discipline in which **the superposition with the laws of the (Euclidean) geometry is straightforward**, because of the rectilinear propagation of the light rays (on short distances)

Descartes' and Newton's evidence in relationship with *Optics*

- **From this intuition, the Newtonian *Geometrical Optics* derives**, so that it is not casual that we find this reference to evidence also in a famous passage of the Newtonian *Treatise on Optics* – quoted also by E. Cassirer as a fundamental source of the Kantian phenomenalism in his monumental treatise on Modern epistemology (Cassirer, 1922, pp. 402-403).
- In this passage Newton exposes **the "turn" from the Aristotelian Science, based on the "explanation by causes", to the Modern Science, based on the "explanation by laws".** "Scientific explanation", from that moment on, means no-longer finding the causes (real relations) of a given, common-sense phenomenon, but finding a time-independent mathematical law (effectively, a functional relation of the calculus) by which making pre-dictable or retro-dictable some given measurable phenomena. This passage, quoted from the original Ancient English used by Newton and that I preserve, reads:

Descartes' and Newton's evidence in relationship with *Optics 2*

- These Principles [the laws of Newtonian Mechanics] I consider, not as occult Qualities, supposed to result from the specifick Forms of Things, but as general Laws of Nature, by which the Things themselves are formed; their Truth appearing to us by Phaenomena, though their Causes are not yet discover'd. For these are manifest Qualities, and their Causes only are occult. And the *Aristotelians* gave the Name of occult Qualities, not to manifest Qualities, but to such Qualities only as they supposed to lie hid in the Bodies, and to be the unknown Causes of manifest Effects (…). Such occult Qualities put a stop to the Improvement of Natural Philosophy, and therefore of late Years have been rejected. To tell us that every Species ["specific essence" or "nature"] of Things is endow'd with an occult specifick Quality by which it acts and produces manifest Effects, is to tell us nothing: But to derive two or three general Principles of Motion from Phenomena, and afterwards to tell us how [not "why"] the Properties and Actions of all corporeal Things follow from those manifest Principles, would be a very great step in Philosophy, though the Causes of those Principles not yet discover'd: And therefore I scruple not to propose the Principles of Motion above-mention'd, they being of very general Extent, and leave their Causes to be found out (Newton, 1730, pp. 376-377)

The calculus at the core of Modern science

- **The connection with Descartes' method is immediate** as soon as we render "manifest", in the text with "evident", as generally made by all the modern versions of this passage – the Cassirer version into German of this text included. However, the connection with Descartes's evidence is much more immediate when we consider more deeply also **the connection with *calculus***.
- To speak about modern mathematics and logic without considering the calculus means that we do not understand nothing about modern science. **What distinguishes absolutely the Modern Age from the Ancient Age is the calculus**, till arriving to the Post-Modern Age that we are rightly connoting as the "Information", and hence the "Computation Age"!

The Newtonian notion of calculus

- Therefore, the connection with Descartes's method is more immediate **when we consider the *functional,* and then the *algebraic* character of the Newtonian Laws of Motion** to which Newton is referring in the quoted passage. That is, all the explicative power of the method consists in finding an algebraic relation (morphism), e.g., a function $y = f(x)$, connecting some values of a dependent variable, with some other values of an independent variable.
- For instance, in the equations of motion in mechanics, it consists in finding the functional relation connecting **the values of different positions in space**, with the values of **different instants of time**.
- If the values **calculated "a priori" correspond with the "a posteriori" values of some effective measurements** (in the case of mechanics, the measured initial and final positions of a moving body in different instants of time**, independently on the direction of the arrow of time,** i.e., the calculated final position can be either in the future, or in the past), this means that we found the mathematical law explaining such a mechanical phenomenon.

An example with the Galilean law

- In the case of the **Galilean equations of motion for a heavy body**, rewritten in a functional way by Newton following his “gravitation law”, we have:

$$z_t = z_0 \pm \tfrac{1}{2} g t^2$$

- Where, z_t is the spatial final position of a heavy body at time $t \neq 0$, z_0 is the spatial initial position at time $t = 0$, g is the gravitational constant, and the +/- sign indicates that the laws of motion of mechanics are invariant for the direction of motion, i.e., for the orientation of the axis of a representation of the integral form of this function over a Cartesian graph.

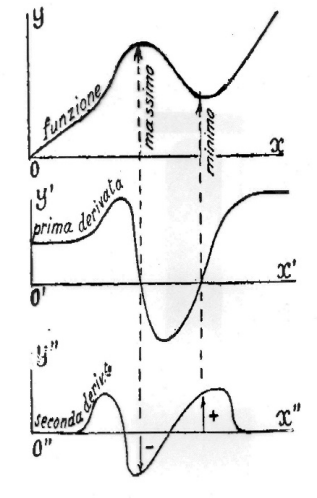
- It is evident that, by cancelling all the constants, we obtain the relation $z \approx t^2$, that is, space is varying as the square of time. I.e., the *geometrical equation* underlying the Galilean law is the well-known **parable binomial equation**, $y = x^2$.

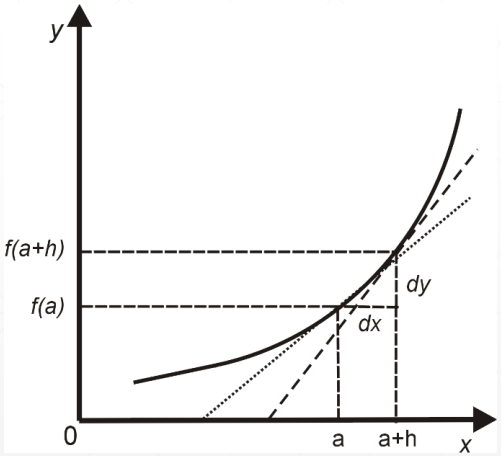
Integrability as the core of calculus

- That is, **a physical law has "evidently" a geometric (algebraic) nature**.
- For the very same reason, the temporal evolution of a mechanic system, i.e., the bodily motion in the space-time, can be geometrically represented as a unidimensional continuous trajectory designed by a "material point" representing the moving body, i.e., **as an integral form, connecting different points corresponding to different spatial-temporal positions** of the body in a Cartesian diagram, having space and time as its axes.
- **This condition of the "integrability" of a function (equation)** – that, as we recalled, is "epochal" because solved the millennial problem of the integral calculus in mathematical physics, unsolved since Archimedes' time –, and characterizing the Newtonian laws of mechanics, **justifies *the refusal of the causal explanation* of the bodily motion, for a reason intrinsic to the method itself**, apart from the metaphysical considerations expressed by Newton in the above quoted text.

Integrability and mechanical vacuum

- Namely, this reason is the “objective” mathematical evidence according to which, ***necessary and sufficient condition*** *for the integrability of the equations of motion in mechanics* ***is the “uniform acceleration”*** *of the body*, that is, the acceleration *must be a constant*, i.e., ***no variation*** **in the second derivative *must* occur.**





Integrability, inertia law and mechanical vacuum

- In other terms, the truth of the First Principle of the Newtonian Mechanics, the Inertia Law, as far as stating precisely the condition that the system is not submitted to any external force or *causal interaction*, i.e., it is completely “isolated”, or it is moving inside the “mechanical vacuum”, **is based on a *mathematical* not physical evidence**, no matter how absurd it might be for the common sense experience the concept of a local motion without friction.
- Roughly speaking, **it is like the “fifth postulate” in Euclidean geometry**.
- Despite **it is contrary to the common sense experience** (two parallel lines (e.g., two rails) in the infinite seem to meet with each other), nevertheless **the necessity of the fifth postulate truth is *rationally* self-evident for justifying all the theorems of the Euclidean geometry**.
- In the same way, because it is possible to perform integral calculations in the Classical (Newtonian) Mechanics, **the inertial principle *must hold,* apart from how much contra-intuitive it might be for the commonsense**. On the other hand, the countless successes of four centuries of Newtonian Science and Technology testify the effectiveness of supposing the *inertia principle* in mechanics.

The apodictic character of Newton's evidence and the scandal of Non-Euclidean geometries

- The supposition of self-evidence of the Newtonian Mechanics principles justifies therefore the apodictic value attributed by Newton to them (his famous ***hypotheses non fingo***), as far as put by Newton, not only at the beginning of his masterpiece, the *Principia*, but also of the *Opticks*. Its first proposition, indeed, reads: "**My Design in this Book is not to explain the Properties of Light by Hypotheses, but to propose and prove them by Reason and Experiments**" (Newton, 1730, p. 1).
- The discovery of **Non-Euclidean Geometries** during the first half of the XIX cent., and **the consequent discoveries of other branches of the physical sciences**, non-reducible to the Newtonian Mechanics, during the second half of the XIX cent., and the first thirty years of the XX cent., determined **the abandon of the *apodictic* method of the early modern mathematical and physical sciences**. They set out the passage to the *hypothetical-deductive* method, proper of the modern mathematical and natural sciences in their adulthood.
- Namely, they are: the Thermodynamics, the Quantum Mechanics, and, finally, the Relativity Theory, both Special and General…

Hypothetical character of modern science from XIX cent. on

- This determined the growing disaffection of modern scientists toward the ***evidence* principle as a sufficient justification of *truth*** in the semantics of modern sciences, i.e., the abandon of the Modern Transcendentalism.
- This led to **the progressive affirmation of the *axiomatic method* in formal logic and mathematics,** from the second half of XIX cent. on, with **the axiomatization of Non-Euclidean Geometries by B. Riemann**, and of the **Euclidean Geometry by D. Hilbert**, passing through the **axiomatization of the arithmetic by G. Peano**, and **of the mathematical logic by G. Frege**, till arriving to **the publication of the *Principia* by A. N. Whitehead and B. Russell**, at the beginning of XX cent.
- The consequent publication, under the determinant influence of B. Russell, of **L. Wittengstein's *Tractatus*,** trying to extend to the analysis of the philosophical language the mathematical logic of the *Principia* (Wittengstein, 1922), **decreed officially the "linguistic turn", characterizing our Post-Modern Age** – even though **theoretically "incomplete", as we know**.

Unreliability of evidence and the “tacit dimension” of knowledge

- Indeed, like such a brief sightseeing of the history of the foundation issues teaches, **the presumed self-evidence of principles in mathematical sciences is effectively *relative* to the different epochs** – think, for instance, at the “fifth postulate” of the Euclidean Geometry in mathematics foundations (Fraenkel, Bar-Hillel and Levy, *Foundations of Set Theory* 85) –, and, more generally, **it is relative also to the different cultures in philosophy**, as it is immediate in our “global” and “multi-cultural” era.
- This weakness of evidence depends, ultimately, on the presence of **what M. Polanyi, following W. Dilthey, defined as the “tacit dimension of knowledge**” (Polanyi & Sen, 2009), as **an unavoidable dimension of any form of *personal knowledge* (Polanyi, 1962)**, and then of any “first-person language”, to say the same thing on the semiotic hand.