Class 16.

Formal Philosophy. The Post-Modern Age:

Language & Naturalism

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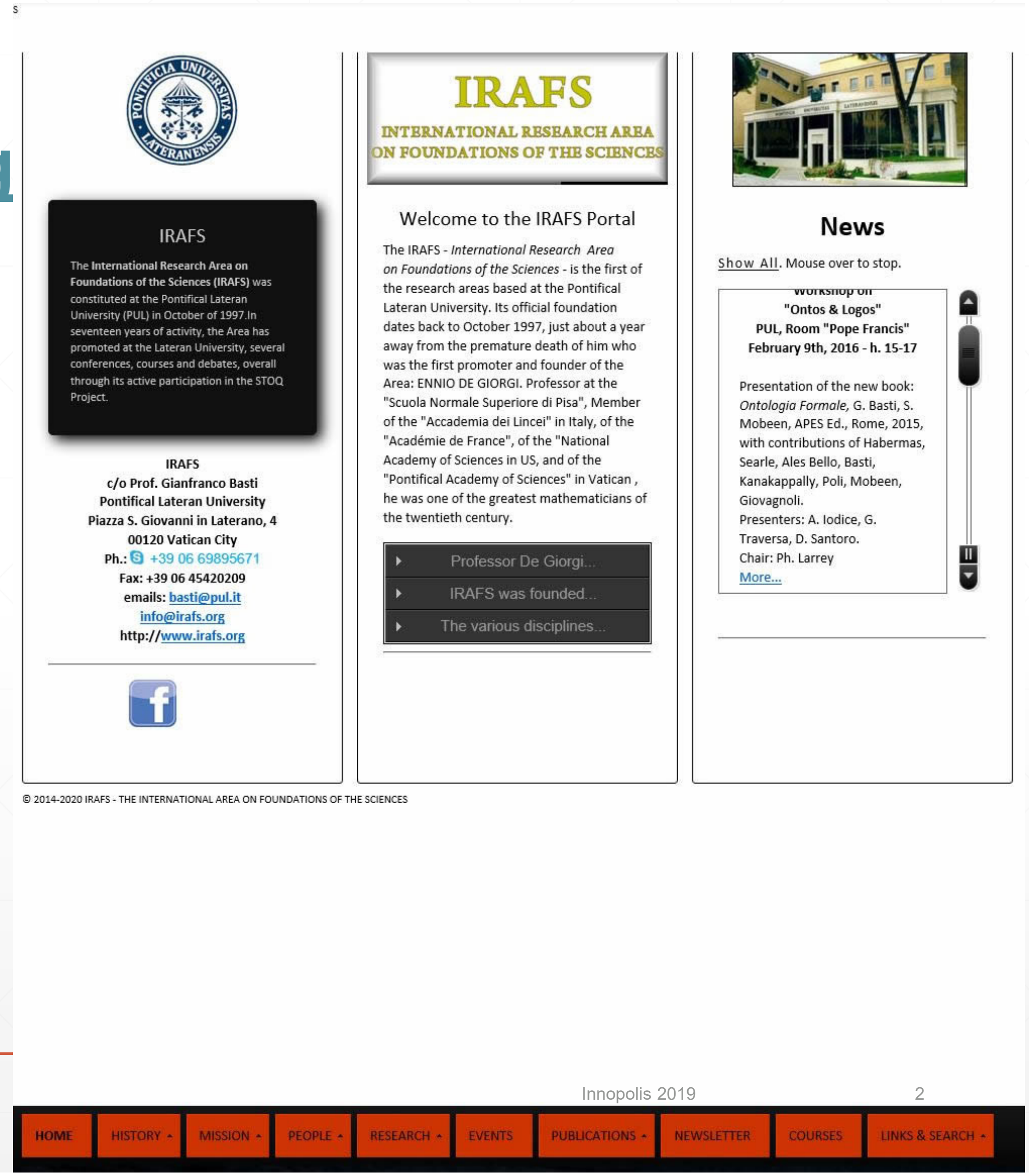
Course: Language & Perception

Syllabus I Part (1-2/11/2019)

Syllabus II Part (8-9/11/2019)



Summary

- The birth of the hypothetical-deductive method because of the discovery of the non-Euclidean geometries and their axiomatization by Riemann's completion of Descartes' initial algebrization of geometry.

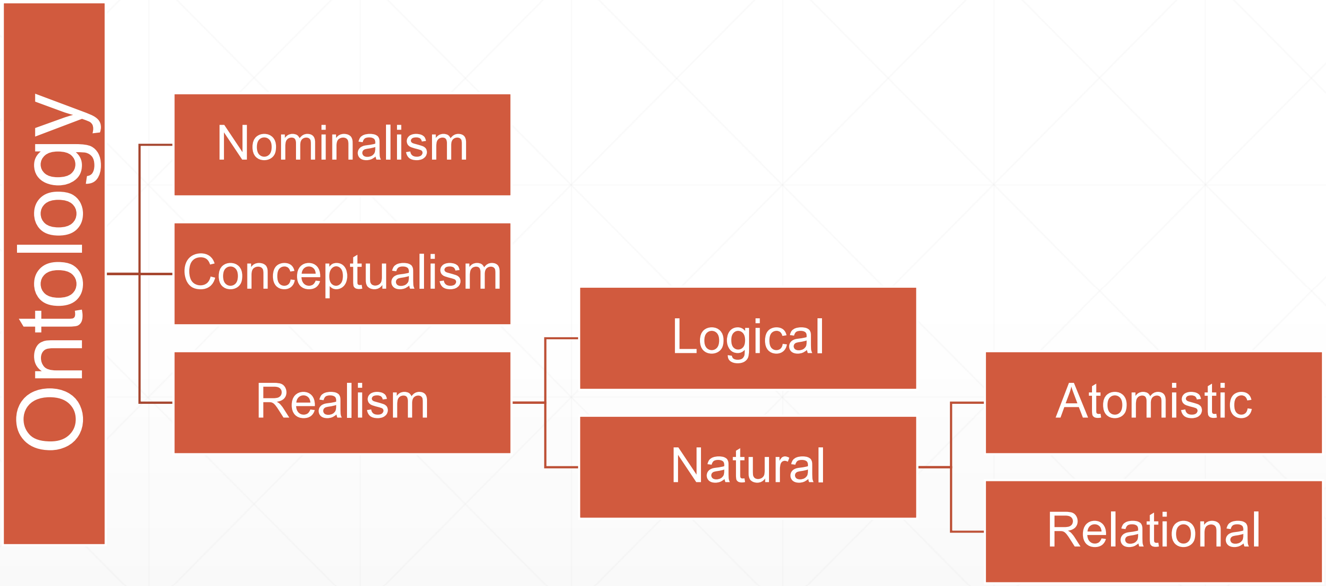
- The abandon of the belief in the apodictic character of mathematics determined, on the one side, the abandon of trusting the cognitivist principle of evidence in epistemology, and on the other side, the necessity of demonstrating the consistency of mathematics, and specifically of calculus, by a proper metalanguage individuated by Weierstrass and Cantor in the set theory.

- Birth of the mathematical logic by Frege's notion of propositional function, and the discovery of the logical antinomies leading to the development of several axiomatic set theories such as ZF, all characterized by the Skolem paradox (their axioms are expressed in first-order logic, but their semantics must be necessarily of a higher order).

- All this determined in the philosophy of science the so-called "linguistic turn" by Wittengstein's and Carnap's logical atomism, with the birth of the neo-positivistic school and its criticism by Popper's "evolutionary approach" to epistemology. Even though such a linguistic turn from modern cognitivism, with its Popperian untenable irrational outcomes, is incomplete until its completion by the semiotic (algebraic) approach.

- <u>Refs.</u>: 3. (chs. 3-4) 10. 17.

Formal Ontologies Scheme



The hypothetical character of the Newtonian physics

- Therefore, the **convergent discovery,** during the XIX cent. of**:**
  1. **The hypothetical character** of mathematics, after the development of the non-Euclidan geometries;
  2. **The development of other branches of physics** whose principles are not directly reducible to the Newtonian three laws of mechanics
- Reduced the **absoluteness pretension** of the Newtonian interpretation of the classical mechanics and of its apodictic method.
- In short, hereinafter, Newtonian mechanics will not be «the mechanics» anymore, but it will identify itself with a subset of it, i.e., **the «classical mechanics»,** which has a well-defined domain of application — the so-called **macroscopic mechanical** phenomena (those of ordinary experience, so to say).

The emergence of several levels of matter organization/study in physical sciences

- At the **mesoscopic** level (molecular aggregates) the principles of statistical mechanics and hence of thermodynamics (linear and non-linear) hold .
- At the **microscopic** level (from the molecule, to the atom, to the sub-atomic and to the sub-nuclear levels) the principles of quantum mechanics and special relativity (quantum electrodynamics and quantum cromodynamics) hold.
- At the **megaloscopic** level, at the level of phenomena at a cosmic scale, the principles of relativistic mechanics (general relativity) hold.
- → Revision of the false, ideological and not scientific pretensions of absoluteness of Newtonian mechanics.

Five new main fields of research in physics I

1. The birth and development of **thermodynamics** as a statistical theory of molecular aggregates that introduces a temporal *irreversibility* in physical phenomena, not compatible with classical mechanics;
2. The birth and development of **quantum mechanics** with its principles of *quantization*, *indeterminacy, exclusion, complementarity*, which have no equivalents in classical mechanics.
3. The birth and development of the **special theory of relativity**, with its fundamental law $E = mc^2$ that shows the reciprocal transformability between mass and energy for particles accelerated to velocities close to the limit-velocity of electromagnetic radiation (light). This is an idea that, combined with the principles of quantum mechanics, has shown to be quite fruitful in the domain of the physics of microscopic, quantum systems, under the form of **quantum electrodynamics** and quantum **cromodynamics.** All these principles are incompatible with classical mechanics.

Five new main fields of research in physics II

4. The **general theory of relativity** which provides an explanation of the force of gravity *G* that Newtonian physics did not have, even if Newton had described its mathematical form, by means of the law of universal gravitation, for the first time in the history of humanity. All this opened the way to the development of the **physical cosmology,** at the beginning only **theoretical,** by developing **mathematical models** of the universe, of its origins and evolution, and from the end of the XX cent., also **observational,** before all by the possibility of ever more precise measurements of the CMBR (cosmic microwave background radiation) → cosmology is now a **Galileian science** in the proper sense of the term.

5. **The new science of complexity** overcoming the old **reductionist interpretation** of nature both in its **syncronic sense** (the emergent levels of complexity in matter organization reduced to their elementary components), and **diacronic sense** (the explation of the behavior of a complex system reduced to the initial conditions of ots dynamics).

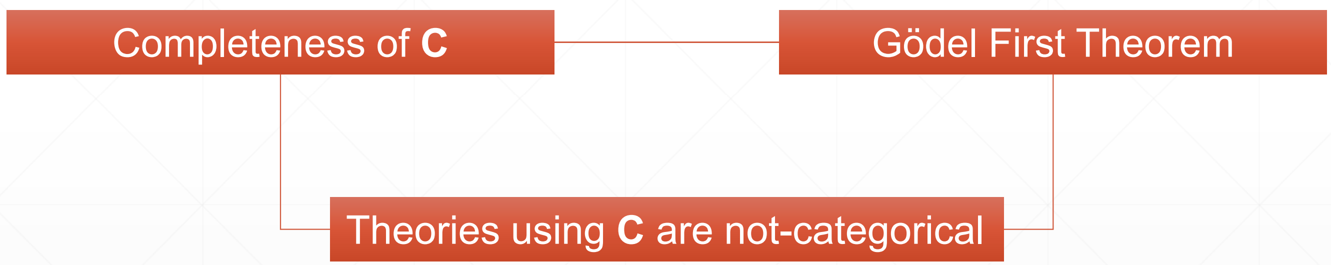
Set-theoretic semantics in the light of Gödel, Skolem, and Dedekind theorems

- There exists a sort of **indetermination relationship** between the demonstrative power and the expressive power of formalized theory depending on a corollary of Gödel incompleteness theorems, even though demonstrated before them (1915-1923), that is the **Löwenheim-Skolem theorem**:
  - «If **a first-order countable (i.e., consistent with the first-order predicate calculus, C)** theory has an infinite model, it is **a countable infinite model**» → any consistent first order theory is **non-categorical,** that is, it has no unique infinitary model **up to isomorphism,** i.e., all its models are **not isomorphic** among them and hence do not belong to the same (**logical) category**.
  - → **Löwenheim-Skolem paradox:** the Cantor set theory and **Cantor theorem** → distinction between **countable infinite sets** (bijective with the set of natural numbers) and **non-countable infinite sets** (the set of real numbers, the set of complex numbers, the set of all subsets of the natural numbers, etc.) cannot be expressed in **any first-order axiomatic set theory,** despite all the axioms of ordinary set theories are **first-oder sentences**.
  - On the other hand, because of **Dedekind theorem** («any infinite set is similar to (can be put in biunivocal correspondence with) a proper part of itself»), it is possible to demonstrate **the completeness of second-order arithmetic.**
  - → Axiomatic set theory as **theory of foundations of mathematics** (like whichever foundation theory) must be formalized using **higher order logic** with its full semantics and hence with **non-countable models** (Zermelo for ZFC, and Von Neumann for NGB both admit the existence of non-countable models only in the second order interpretations of ZFC and NGB set theories, respectively) → **finitary character** of any first-order theory.

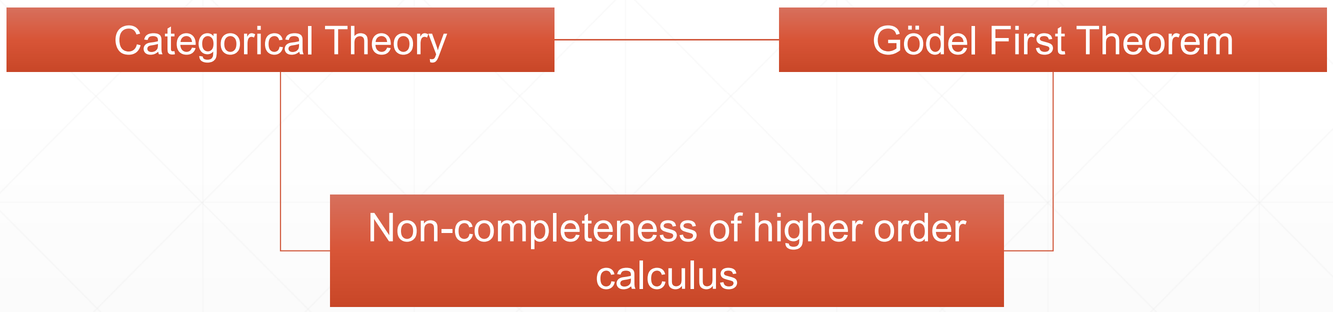
The trade-off between demonstrative power and categorical power of a formalized theory in the light of Gödel, Skolem, and Dedekind theorems

- → **Indeterminacy relationship consistency/categoricity (De Giorgi):**
  - If a formalized theory is based on **a powerful demonstrative method** (first order predicate calculus, **C**), it loses **explicative unity,** i.e., it is **not categorical** (e.g., first order theories).
  - If a formalized theory is **categorical,** i.e., endowed with a strong **unitary explicative power,** it is based on **a weaker demonstrative method** (higher order predicate calculus).
- **To sum up:**

Completeness of the calculus *versus* the categorical character of a formalized theory I



Completeness of the calculus *versus* the categorical character of a formalized theory II



The Skolem paradox and the relativity principle in the algebraic interpretation of set theory

- In (Skolem 1922) Skolem himself gave an interpretation of the **relevance** of his theorem for the axiomatic set theory according to his **algebraic or model-theoretic interpretation** of it:
  1. The axiomatic set theory **cannot be a suitable theory of foundations of mathematics** since essential notions such as non-countable sets cannot be formalized in it.
     - Effectively this interpretation is not true because it is sufficient to use a **second-order semantics** in axiomatic set theory for granting the existence of **non-countable models** to axiomatic set theories such as ZFC and NGB → **Skolem paradox**, from the mathematical logic standpoint is only interesting as revealing «a novel and unexpected feature of formal systems» (van Heijenoort 1967) with no unavoidable antinomy for set-theoretical foundations of mathematics.
  2. The notions of **countable/non-countable sets** in any algebraic or first-order model-theoretic interpretation of set theory is **relative to the algebra of sets** we are using. This interpretation is **true in two senses:**
     - What the Skolem theorem forbids to set theoretic semantics is an **absolute representations of non-countable objects** such as real numbers, in the sense that the set of **all real numbers** is not countable, but this does not imply that **subsets of them** can be posed in a bijective relation with natural numbers (Bays 2014), as effectively the same **diagonal method,** used by Cantor for demonstrating that the full set of them is non-countable, demonstrates.
     - It is possible to develop an **object-free category theory** and then an **ante-predicative theory of categories,** in which the same **objects** x, even **set elements** constituting the domains of logical predicates, must be conceived **as domains-codomains of relations** and corresponding to **as-many identity (reflexive) relations** Id_x → **relational semantics of Tarski and Kripke in mathematical and modal logics respectively.**