Class 12.

Beyond Cognitivism I
The Peirce Lesson of Semiotics

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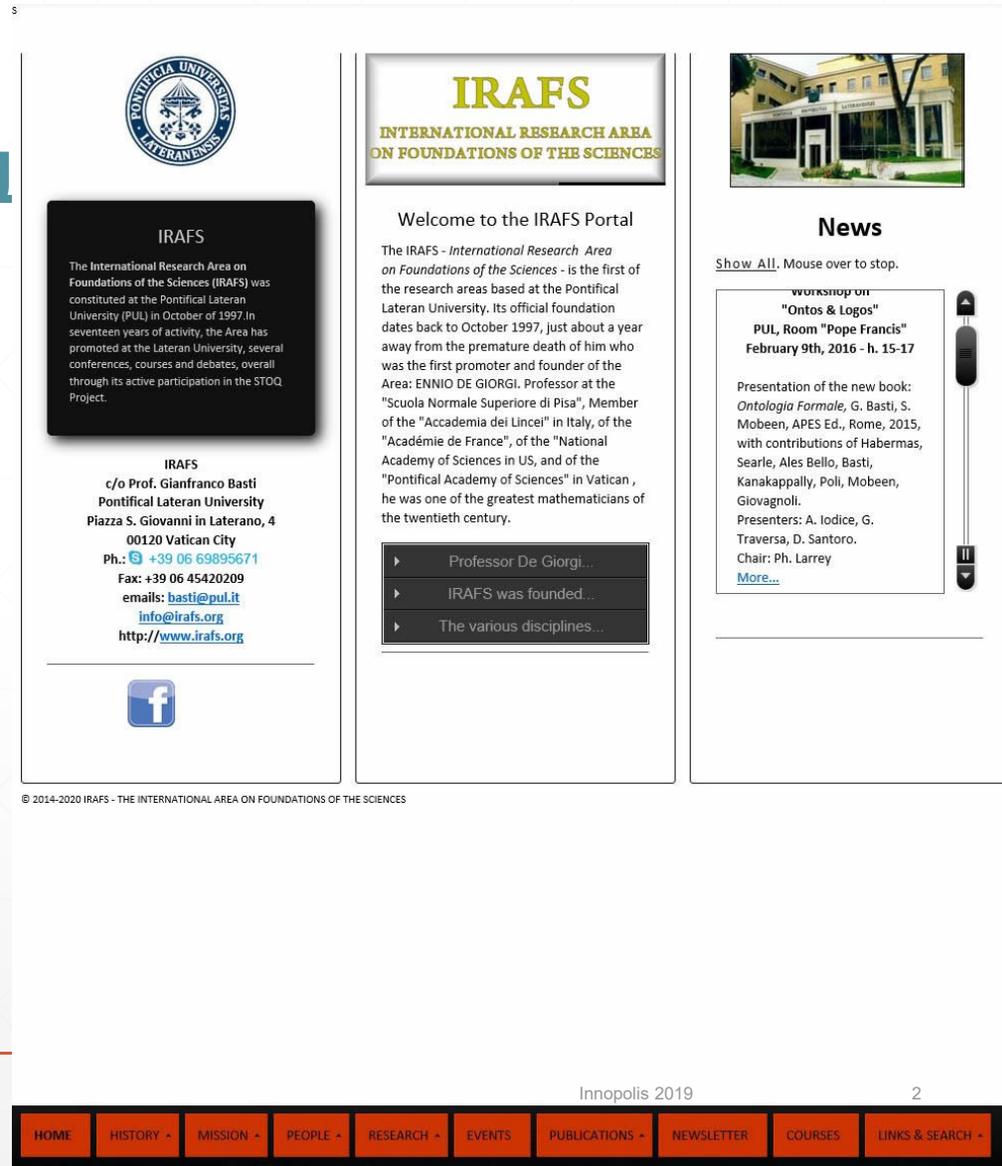
Course: Language & Perception

Syllabus I Part (1-2/11/2019)

Syllabus II Part (8-9/11/2019)

Summary

- The **algebraic origins of modern mathematical logic in Boole's and Schröder's work** and the convergent criticism to the purely syntactic formalism of Schröder's approach by Husserl – from the cognitivist standpoint – and by Peirce from an algebraic standpoint, vindicating **the triadic character** of every signifying algebraic structure underlying any predication in logic.

- This constitutes the core of his interpretation of logic as **"formal semiotics".**

- This led Peirce to the development of his famous **ante-predicative theory of algebraic categories** (**firstness-secondness-thirdness**) as underlying any predicative theory of categories in logic, as well as to the development of his pioneering algebraic theory of mathematical logic that was contemporary to Frege's mathematic logic based on his logic of classes.

- This invention by Peirce of the **algebra of relations** remained however not valorized till Tarski's axiomatization of the logic of relations into **a calculus of relations** during the second half of XX cent., including the axiomatization also of the algebraic notion of category, underlying the actual development of the **Category Theory** as metalanguage of logic and mathematics in many senses wider than standard set theory.

- Refs.: 5.

Formal Ontologies Scheme
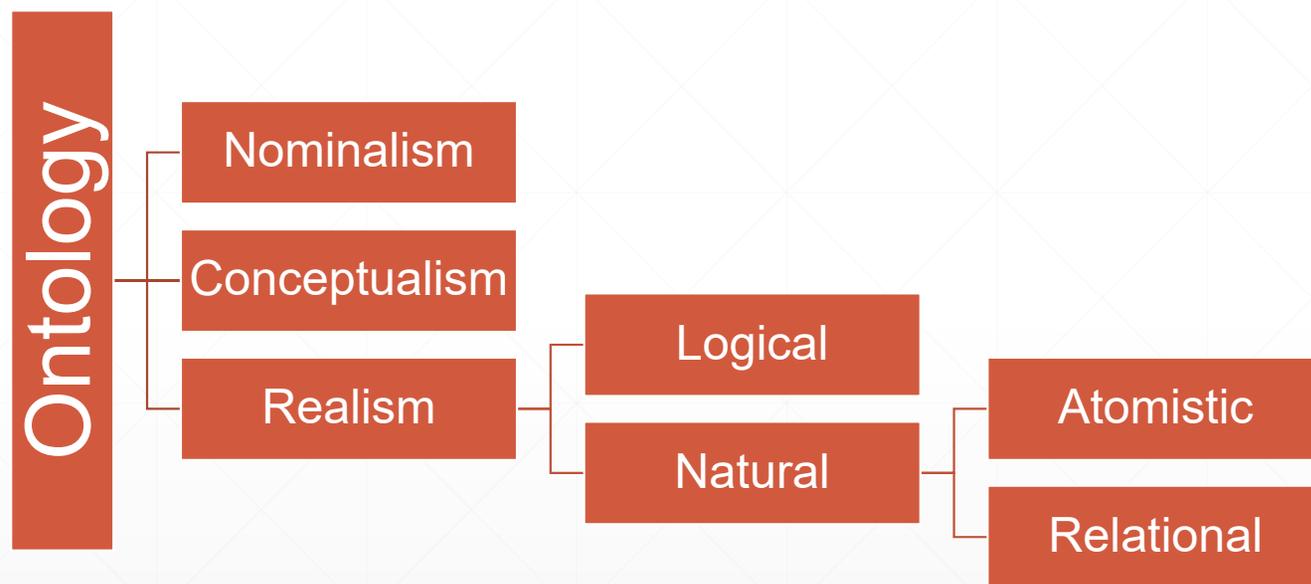
The semiotic completion of the linguistic turn

▪ Generally the **post-modern «linguistic turn»** is related with the works of Gottlob Frege and Ludwig Wittengstein, as depending on the abandon of the evidence principle as foundation of truth beacuse of the birth of the **axiomatic method** after Riemann and the development of the **non-Euclidean geometries.**

▪ Frege by the notion of **propositional function** (a formula that contains variables) unifying the Aristotelian **predicate logic** ($P(x)$) with the Stoic **propositional logic** ($p \wedge q$) in one only propositional and predicate calculus, was still linked to a **cognitivist prejudice:** the notion of class depends on the unifying act of the intellect by his so-called **axiom of comprehension:** $\forall x \exists y : x \in y \leftrightarrow \varphi(x)$.

▪ Wittengstein also by his notion of **atomic proposition** ("the snow is white") dependning on an **elementary perception** of a world fact or state-of-affairs was linked to a **cognitivist prejudice.**

▪ The completion of the linguistic turn only with Peirce and his proposal of formal logic as **formal semiotics.**

Peirce's and Husserl's inquiry over the pre-logical foundation of signifying

- The possibility of developing an algebraic foundation of predication in logic is the main result of the actual development of **Category Theory** (CT) as a universal meta-language of logic and mathematics.

- Its relevance is due to the development of the **operator algebra formalism** in any field of physics starting from quantum physics and in computer science, before all in computational linguistics, as we discuss in other parts of this course.

- In fact, CT consists in an **ante-predicative, algebraic and hence pre-logical Category Theory,** even though its extreme versatility allows it to express in its formalism also the standard model-theoretic and hence **logical** theory of category to which Gödel and Skolem theorems refer (Heller 2007).

- This result was philosophically anticipated at the end of the XIX cent. by **Husserl's and Peirce's <u>convergent</u>, but <u>independent</u> criticism** to Schröder *Algebra of Logic* that pursued the aim of the foundations of mathematical logic by an algebraic set-theory of countable sets, and of which the **Löwenheim-Skolem Theorem** demonstrated the inconsistency in 1921:

    - **Convergent** because both Husserl and Peirce vindicated that any foundation theory of logic and mathematics must justify before all the same notion of **signifying in the constitution of the same domains of predicates/functions,** and this can be obtained only by **referring to a third term** beside the dyadic relationship subject/predicate (argument/function): **the interpreter/interpretant.**

Continuing…

- **Independent** because:

  - <u>Husserl</u> affirmed that this third term must be necessarily an **interpreter,** i.e., a **knowing transcendental subject,** even though **an intentional subject** constituting the **objects** of the consequent logical/mathematical predication.

  - <u>Peirce</u> invented the neologism **interpretant,** just for affirming that this third term making **signifying because triadic** the dyadic subject-predicate (argument/function) relation is independent form any knower → irreducible character of the triadic relations (Tarski) in the algebra of logic + **Peirce's pre-logical (semiotic) theory of categories** (firstness, secondness, thirdness) in its interpretation of formal logic as formal semiotics.

- Let us deepen the Peirce contribution.

The relational semantics as the core of Peirce's semiotic completion of the linguistic turn

On the Definition of Logic. Logic is *formal semiotics.* A sign is something, *A*, which brings something, *B*, its *interpretant sign*, determined or created by it, into the same sort of correspondence (or a lower implied sort) with something, *C,* its *object,* as that in which itself stands to *C.* This definition no more involves any reference to human thought than does the definition of a line as the place within which a particle lies during a lapse of time. It is from this definition that I deduce the principles of logic by mathematical reasoning, and by mathematical reasoning that, I aver, will support criticism of Weierstrassian severity, and that is perfectly evident. The word "formal" in the definition is also defined (Peirce, *The New Elements of Mathematics* 54).

▪ → Peirce's invention of the **Algebra of Relations:** *Logic of Relatives* (1897), a review of Schroeder's book *Algebra of Logic*: the fundamental irreducible relations in algebra and logic are not the **dyadic ones, but the triadic ones.** Roughly speaking, subject and predicate (i.e., a dyadic relation: the monadic predicates of Boolean Algebra) **signify as far as they both are in a mirrored relationship to a third one,** *the interpretant* **(not "interpreter" even though it is not excluded).**

The relation with interpretant as homomorphism

- It is relevant for our aims that in the following passage of Peirce, in which he introduces for the first time the notion and the term of "interpretant", there is a sort of ancestor of what in CT is the notion of "meaning function" as **homomorphism**, that is, a **structure-preserving mapping** between two algebraic structures.

- For understanding this, it is sufficient to recall that "map" in algebra is synonym of "representation" without any reference to a knowing subject:

  - "Every comparison requires, besides the related things, the qualitative ground (= predicate) and the correlate (= subject), also a mediating representation which represents the relate to be a representation of the same correlate which this mediating representation itself represents. Such a mediating representation may be termed an "interpretant", *because it fulfils the office of an interpreter who says that a foreigner says the same thing which he himself says"*.

Peirce's invention of the algebra of relations

- On this **triadic algebra of relations** is based, therefore, the semiotic notion of *sign* as a "being for" a third term, by which the dyadic relation of "being to" between the two terms of a predicative relation, acquires the capacity of signifying. On this theory, also Peirce's famous theory of the three *semiotic categories*, "firstness", "secondness", "thirdness" is based.

- These are **ante-predicative categories**, in the sense that any classical predicative theory of categories (i.e., intended as the most general and then irreducible predicates in a given language), supposes this algebraic triangulation these three semiotic categories.
  - See the **commuting diagrams** in algebra of relations as the fundamental tool granting uniqueness and then universality to its demonstrations, and where the triangular one is the most fundamental.

- This implies that this theory of the semiotic categories, acquires its proper meaning and historical relevance only in the context of Peirce's original contribution to the algebra of logic.

- In fact, till Peirce no *algebra of relations* existed, given that from Aristotle, to Boole, De Morgan and Schröder, any relation was only dyadic.

- Effectively, an ancestor of Peirce John Poinsot (XVII sec) anticipated Peirce speaking about the intrinsic **triadic structure of the signifying relation** in languages saying that this relation does not belong to the category of **dyadic relations (*esse ad,*** being to) because it needs a **third term (*esse per,*** being for).

Poinsot's proto-semiotic (Deely) interpretation of Aquinas' ontology of truth: *conformitas* proposition-thing
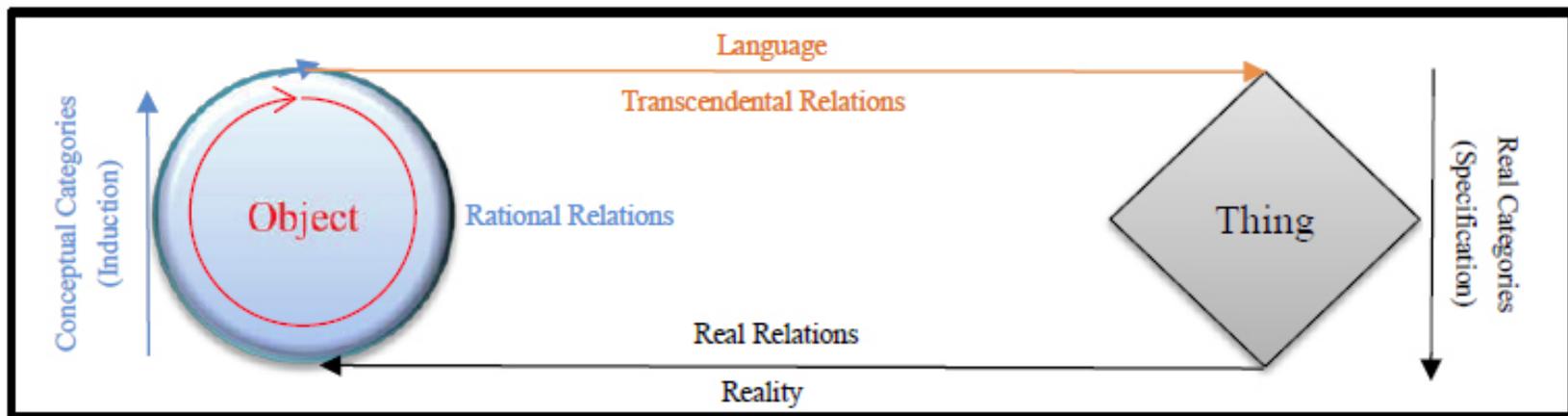
"The *transcendental relation*, that is nothing but a relation in language, has not the principal meaning of "relation" [i.e., it does not belong to the ontological category of "relations"], but of something "absolute" to which some relation can be attributed [e.g., a rational relation, or a real relation]. Indeed, if it was not implying something "absolute", it would be not "transcendental", that is ranging over different categories (*idest vagans per diversa genera*), but it would belong to only one category (Poinsot, *De Signis,* 578b5-579a7. In: (Deely, *Tractatus de Signis. The Semiotic of John Poinsot* 90)).