Uniwersytet
Kardynała Stefana Wyszyńskiego
w Warszawie

Towards a Contemporary Ontology

The New Dual Paradigm in Natural Sciences: Part I

Module 2 Class 1: The Greek origins of Western science

Course WI-FI-BASTI1

2014/15

Introduction

Class 1: “The Greek origins of Western science”

Course modules

Modules	Topic	Suggested Readings
	SECTION ONE	
0.	*Introduction and Course Overview*	
1.	The Birth of Modern Science	Refs.: **1**, chs. 0, 1, 2.
2.	The Question of Truth in Modern Science	Refs.: **1,** chs. 3, 4.
	SECTION TWO	
3.	The Information Theoretic Interpretation of QM	Refs.: **4-10**.
4.	QFT interpretation as 'Second Quantization' and the Physics of the Condensed Matter	Refs.: **11**, pp. vi-xii, 1-35, 137-178; **12; 13**.
	SECTION THREE	
5.	The (Co)Algebraic Interpretation of QFT as q-deformed Hopf Algebra / Coalgebra	Refs.: **11,** pp. 131-185; **14**.
6.	The DDF Principle of QFT, its Cosmological Relevance and Its Ontological Interpretation	Refs.: **14-19**; **1**, ch. 5.
	SECTION FOUR	
7.	Universal Coalgebra and the Interpretation of QFT Systems as STS	Refs.: **16**; **20**
8.	*Conclusions*	

Main Contents of the Module 2

- This module 2 is mainly concerned with the problem of **truth** and/or the **problem of foundation** in pure and applied modern mathematical sciences**.** A problem emerging from the discovery of the **hypotetical nature of mathematics,** from which the, necessity derives of proving the consistency of mathematics – geometries and arithmetics, before all – given that the validity of the demonstrations is no longer granted by the soundness of the axioms. A particular subset of this more general problem concerns the **soundness** of the mathematical laws of physics.

- The four classes of this module concern**:**

1. **The Greek origins of the Western science**.
2. **The issue of the V postulate of Euclidean geometry in the history of Western mathematics**
3. **The issue of the foundations of mathematics from Riemann to Gödel and the Hilbert second problem**
4. **The issue of the Skolem paradox and the relativity principle in the algebraic interpretation of set theory**

Bibliography

Bibliography of the Class 1 of the Module 2

Bibliography

- Main References for this Module

  1. Gianfranco Basti, *Philosophy of Nature and of Science. Volume I: The Foundations,* transl. by Philip Larrey, Rome 2012 (for student use only), ch. 0,1,2, 3 [attached].
  2. C. B. Boyer, *A history of mathematics,* J. Wiley & Sons, New York, 1968 (available online at: https://archive.org/details/AHistoryOfMathematics (*Second Edition, ed. by U. C. Merzbach,* J. Wiley & Sons, New York, 1991).

- Other Reference:

  - E. Nagel, J. R. Newmann, *Gödel's proof. Revised edition,* Ed. by D. R. Hofstadter, New York UP, New York, 2001 (1958 First edition available online at the Internet Archive of the University of Florida: https://archive.org/details/gdelsproof00nage).
  - M. Hallett, *Cantorian set theory and limitation of size,* Clarendon Press, Oxford, 1984.

Class 1

The Greek origins of Western science

The Greek origins of the mathematical physics

- As we know, the distinction between **science of nature** and **philosophy of nature** started during the 16th and 17th Centuries, with the **phenomenalism** of Newtonian interpretation of the Galilean science, characterized by its specific *object* (natural phenomena) and *investigative method* (experimental measurements), as well as with its own *formal language* (mathematical) and *demonstration method* (initially apodictic-deductive and later hypothetical-deductive), completely different than the object, method and language of ancient metaphysics and natural philosophy.
- We know also that, on the other hand, the initial **essentialist** interpretation of the Galilean mathematical science of nature had its ancient roots in Pythagoras' (6th century B.C.) doctrine, which made mathematical entities the essence of physical reality, was used by Plato (4th century B.C.), developed by the exceptional axiomatic work of Euclid of Alexandria (3rd century B.C.) and systematically applied to the study of physical realities and their laws by Archimedes of Syracuse (3rd century B.C.).

The early Greek development of the axiomatic method in Aristotle and Euclid

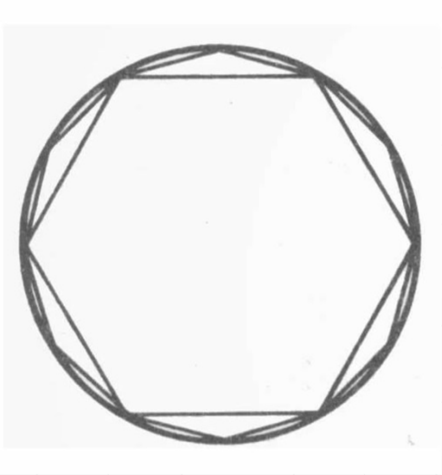
- If Plato’s essential contribution to Western science was his **analytic method** making of Plato’s Academy the cradle of the Hellenistic mathematics**,** Aristotle’s criticism to the Platonic doctrine of the **formal participation of ideas** to the physical reality (inconsistencies of the formal derivation of **duality** from **unity):**
  1. → development of Aristote’s notion of the **abstract, mental nature** of the mathematical entities → his denotation of «the science of being» as **metaphysics** because founded on physics and not on mathematics;
  2. → development of Euclid theory of **prime numbers** → his first formulation the so-called **fundamental theorem of arithmetics** («each natural number is the result of the product of a finite numbers of primes»).
- → Transformation of the Platonic **metaphysical principle of formal participation** into the **logical principle of formal inference** → development of the first two examples of **axiomatic systems** in the history of thought:
  1. **Aristotelian syllogistic system** as the first example of systematization of the **formal logic** based on the propositions of the ordinary language and not on those (more rigorous) of the mathematical language;
  2. **Euclidean books of *Elements*** as the first example of systematization of the **formal mathematics** (arithmetics and geometry) as a rigorous deductive discipline independent from both physics and metaphysics.

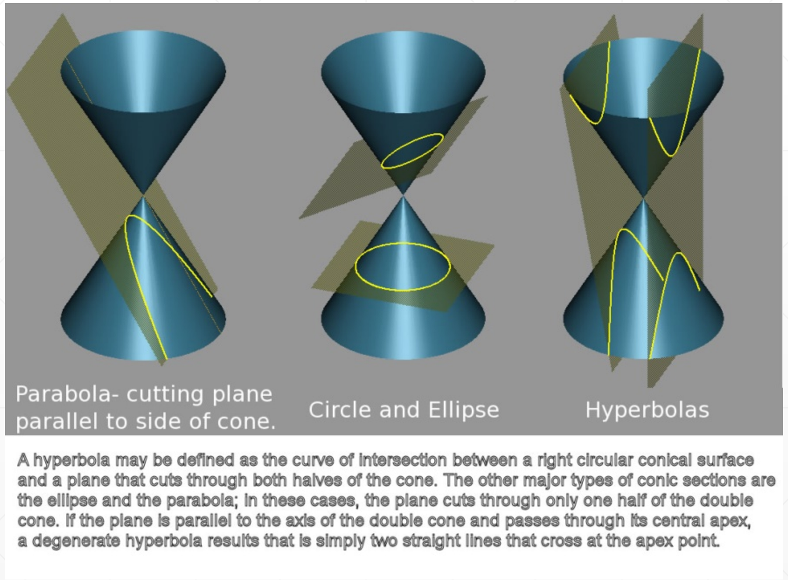
Eudoxus' and Archimedes' contributions and the birth of the problem of calculus I

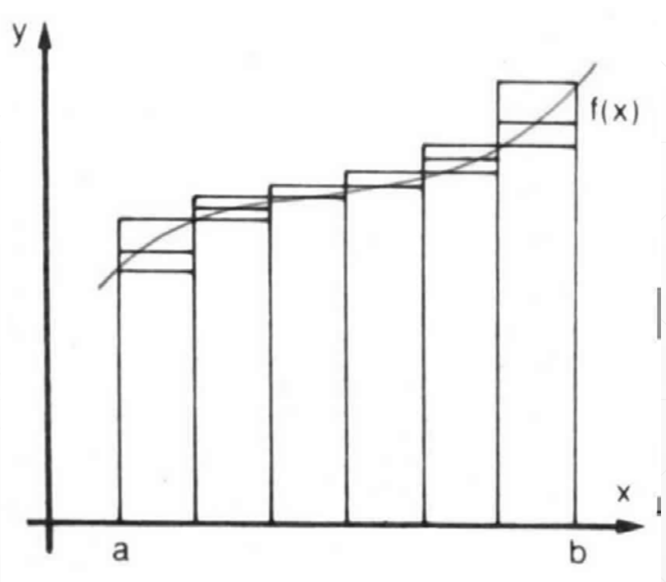
- The use of the empirical method in mathematics and particularly in geometry returns with **Archimedes of Syracuse** (287-212 BC), but as a **heuristic method** for the discovery of new theorems, about which a formal demonstration would be provided, usually through a ***reductio ad absurdum*** **proof** he generalized from the **exhaustion method** applied by another talented disciple of Plato: **Eudoxus of Cnidus** (408–355 BC) – the greatest mathematician of the Hellenistic age, whose work Archimedes is the main testimony – to his **demonstration of existence of the infinitesimals,** in order to solve the problem of the **incommensurable quantities** (i.e.**. Irrational numbers)**
  - By the joint work of these two geniuses → some of the **deepest results** of ancient mathematics derive, such as the calculus of the area and of the volume of a sphere, of the cylinder, of a parabolic curve, as well as the very value of π.
- Waiting for the development of modern **infinitesimal calculus** by Newton and Leibniz, Archimedes' used the so-called **mechanical method** by using mechanics theorems (overall his theorems on the mechanical balance) for calculating geometrical magnitudes so to become the precursor of the modern **analogue computing.** Particularly, he applied this method for calculating the area/volume delimited by conic curves.

Eudoxus' and Archimedes' contributions and the birth of the problem of calculus II

- Intuitive representations of the **exhaustion method** (left)**,** the **conic curves** (middle)**,** and of the **squaring** of non-conic curves of any shape, in its modern functional representation (right).







Eudoxus' essential contributions I (Boyer 1968, 98-102)

- Eudoxus was certainly the greatest mathematician of the Academy school because he helped Greek mathematics to overcome its *horror infinity* consequent to the discovery of irrationals, even though we have no writings from him. His essential contributions were:

1. His very abstract, but rigorous definition of **ratio between magnitudes**, reported by Euclides, source of all his other discoveries *(Elements:* Def.5, Book V)
   - "Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and the third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or are alike less than, the latter equimultiples taken in corresponding order."
     - That, is $a/b = c/d$ iff, given the integers m and n, whenever $ma < nb$, then $mc < nd$; or if $ma = nb$, then $mc = nd$; or if $ma > nb$, then $mc > nd$. It is evident that such a definition is not far from the classical definition of **real** number because it separates the class of rational numbers m/n into two categories according as $ma \leq nb$ or $ma > nb$, so to give a rigorous basis for satisfactory proofs of theorems involving proportions.

2. His **lemma of continuity** (attributed to Archimedes), i.e.:
   - "Given two magnitudes having a ratio (that is, neither being zero), one can find a multiple of either one which will exceed the other."

Eudoxus’ essential contributions II

3. From the lemma of continuity it easy to derive, by *reductio ad absurdum* the proposition defining the **exhaustion property:**
   - “If from any magnitude there be subtracted a part not less than its half, and if from the remainder one again subtracts not less than its half, and if this process of subtraction is continued, ultimately there will remain a magnitude less than any preassigned magnitude of the same kind.”
   - This proposition is equivalent to the modern statement that if M is a given magnitude, ε is a preassigned magnitude of the same kind, and r is a ratio such that $1/2 \leq r < 1$, then we can find a positive integer N such that $M(1-r)^n < \varepsilon$ for all positive integers $n > N$, that is, using the modern notion of limit: $\lim_{n\to\infty} M(1-r)^n$.

- On this basis Archimedes stated that Eudoxus was able to prove theorems about the **areas and volumes of curvilinear figures**, such as the volume of the cone is one-third the volume of the cylinder having the same basis and altitude, the area of circle and the volume of sphere (See *Elements* XII, 2).
- → Eudoxus as «the apparent originator of the integral calculus, the greatest contribution to mathematics made by associates to the Platonic Academy» (Boyer).

Archimedes' essential contributions I (Boyer, 1968, 134-156)

- By his two treatises *On the equilibrium of planes* and *On floating bodies* defining respectively his famous mechanical laws on levers and on flotation Archimedes effectively invented **mathematical physics** based on the **geometrical laws** of mechanics, based on **measurable observations**, deeply different from Aristotle's *Physics* treatise, based on the **causal laws** of dynamics based on **common sense observation.**

- Another fundamental and famous contribution in the realm of geometry was his calculation, using the exhaustion method, **of the value of π** as:
 $3x10/71< \pi >3x10/70$.

- Moreover, as to conic curves, in his treatise *On the quadrature of the parabola (orthotome)* he was able to calculate the area delimited by a **parabolic segment** even though was not able to calculate the area of a general segment of an **ellypse** or of a **hyperbola.** Effectively, in modern terms, while the integration of the first one requires no more than polynomials, the second ones – as well as **the arcs** of these curves or of the parabola – require transcendental functions.

- Nevertheless in the other treatise *On conoids and spheroids* Archimedes found the area of the **entire ellipse:** «the areas of ellipses are as rectangles under their axes» (Prop. 6) and showed how to calculate the volumes generated by the revolution about the principla axis of segments cut from an ellipsoid, a paraboloid, and an hyperboloid.

Archimedes’ essential contributions II

- Apart from his other important contributions of advanced mathematics deriving from A. treatises *On spirals, On the sphere and cylinder,* and *On lemmas* it is essential for us the treatise *The Method* that was re-discovered only in 1906.
- In it Archimedes not only admitted the lack of rigor of his method in supposing, for instance, that **an area is the sum of line segments,** but he anticipated also **two fundamental components** of the modern discussion on the foundations of calculus, and hence of modern mathematical phsyics. Indeed he vindicated:
  1. That the effectiveness of Eudoxus exhaustion method he systematically used for obtaining all his main results **depends critically on preliminary assumptions**, i.e., in modern terms, he affirmed **the non costructive character** of the *reductio ad absurdum* proofs, (e.g., for the existence of limits);
  2. That he obtained his main results both in mechanics and geometry (e.g., the lever laws or the calculus of the area of parabolic segment), «**by balancing lines as one balances weights».** I.e., in modern terms he affirmed in a seminal way, the strict dependence of the applicability of calculus in physics, on the supposition of the **equilibrium stability of classical mechanics,** as the Newtonian laws and particularly, the third one, emphasize.
- Only at the price, indeed, of recognizing a paradigmatic value of Newtonian mechanics – as we see in the fourth class of this module, by discussing the contemporary destiny of Hilbert’s «sixth problem» in front of QFT challenges – is possible to continue to pursue the modern aim of **reducing dynamics to kinematics** and **thermodynamics to statistical mechanics.**

Enrico Fermi’s application of Archimedes’ method

- The former students of **Enrico Fermi** tale an anecdote confirming the effectiveness of Archimedes’ «mechanical method».
- When Fermi was professor at the Institute of Physics of the Faculty of Engineerring of the University of Rome, located in the famous site of **Via Panisperna**, before migrating to US at the Los Alamos lab, he had to calculate the thickness of the paraffin wall, necessary for stopping the chain reaction of his famous experiment, performed within the basin for the red fishes (without them, of course) of the fountain in the faculty garden – evidently, the gap of resources for the basic research between Italy and US was wide also at that time!
- He asked then the mathematicians of the Faculty of Mathematics for the solution of the integral, necessary for calculating precisely the required value.
- Because they delayed in giving him the answer, he designed the integral form on a sheet metal, trimmed it off, and **weighed it,** so to obtain the approximate value, sufficient for performing successfully the experiment!
- Archimedes occupies thus an eminent stall in the history of XX cent. quantum physics (and not only of physics, given the utilization of such a result at Los Alamos)!