Uniwersytet
Kardynała Stefana Wyszyńskiego
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Towards a Contemporary Ontology

The New Dual Paradigm in Natural Sciences: Part I

Module 2 Class 2: The issue of the V Postulate

Course WI-FI-BASTI1

2014/15

Introduction

Class 2: “The issue of the V postulate of Euclidean geometry in the history of Western mathematics”

Course modules

Modules	Topic	Suggested Readings
	SECTION ONE	
0.	*Introduction and Course Overview*	
1.	The Birth of Modern Science	Refs.: **1**, chs. 0, 1, 2.
2.	The Question of Truth in Modern Science	Refs.: **1,** chs. 3, 4.
	SECTION TWO	
3.	The Information Theoretic Interpretation of QM	Refs.: **4-10**.
4.	QFT interpretation as ‘Second Quantization’ and the Physics of the Condensed Matter	Refs.: **11**, pp. vi-xii, 1-35, 137-178; **12; 13**.
	SECTION THREE	
5.	The (Co)Algebraic Interpretation of QFT as q-deformed Hopf Algebra / Coalgebra	Refs.: **11,** pp. 131-185; **14**.
6.	The DDF Principle of QFT, its Cosmological Relevance and Its Ontological Interpretation	Refs.: **14-19**; **1**, ch. 5.
	SECTION FOUR	
7.	Universal Coalgebra and the Interpretation of QFT Systems as STS	Refs.: **16**; **20**
8.	*Conclusions*	

Main Contents of the Module 2

- This module 2 is mainly concerned with the problem of **truth** and/or the **problem of foundation** in pure and applied modern mathematical sciences**.** A problem emerging from the discovery of the **hypotetical nature of mathematics,** from which the, necessity derives of proving the consistency of mathematics – geometries and arithmetics, before all – given that the validity of the demonstrations is no longer granted by the soundness of the axioms. A particular subset of this more general problem concerns the **soundness** of the mathematical laws of physics.
- The four classes of this module concern**:**

1. **The Greek origins of the Western science**.
2. **The issue of the V postulate of Euclidean geometry in the history of Western mathematics**
3. **The issue of the foundations of mathematics from Riemann to Gödel and the Hilbert second problem**
4. **The issue of the Skolem paradox and the relativity principle in the algebraic interpretation of set theory**

Bibliography

Bibliography of the Class 2 of the Module 2

Bibliography

- Main References for this Module
  1. G. Basti, *Philosophy of Nature and of Science. Volume I: The Foundations,* transl. by Philip Larrey, Rome 2012 (for student use only), ch. 0,1,2, 3 [attached].
  2. C. B. Boyer, *A history of mathematics,* J. Wiley & Sons, New York, 1968 (available online at: https://archive.org/details/AHistoryOfMathematics (*Second Edition, ed. by U. C. Merzbach,* J. Wiley & Sons, New York, 1991)*.*
- Other Reference:
  - I. Toth, *Aristotele e i fondamenti assiomatici della geometria. Prolegomeni alla comprensione dei frammenti non-euclidei nel "Corpus Aristotelicum" nel loro contesto matematico e filosofico. 2nd revised and corrected ed.,* Vita e Pensiero, Milano, 1998.
  - E. Nagel, J. R. Newmann, *Gödel's proof. Revised edition,* Ed. by D. R. Hofstadter, New York UP, New York, 2001 (1958 First edition available online at the Internet Archive of the University of Florida: https://archive.org/details/gdelsproof00nage).
  - M. Hallett, *Cantorian set theory and limitation of size,* Clarendon Press, Oxford, 1984.

Class 2

The issue of the V postulate of Euclidean geometry in the history of Western mathematics

The centrality of Euclid's *Elements* in the Western tradition

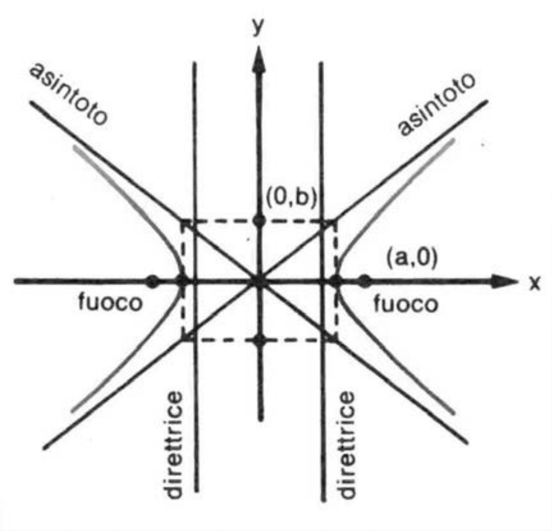
- Differently from Archimedes' books that often get lost and then reappeared, during the more than 2000 years of their history, the almost contemporary Euclid's *Elements* **were objects of constant study and interpretation,** like Aristotle's two books of the *Analytics.*
  - They constitute indeed two monuments of the Greek origins of the Modern **axiomatic method in mathematics and logic**, and testimony of the most fruitful heritage of Platonic philosophy, with its discovery of **logical and mathematical universals.**
  - The axiomatic method, indeed, is what – till now, where the globalization cancelled such a difference – distinguishes the Western thought from the Eastern one, explaining **why science developed in the Western tradition**, despite mathematics and logic **originated in the Eastern tradition** – the Chinese and Indian ones, before all

Euclid’ Five Postulates

- The five Euclid’s postulates of his geometry in the *Elements* I book read:
  - 1. To draw a straight line from any point to any point.
  - 2. To produce a finite straight line continuously in a straight line.
  - 3. To describe a circle with any centre and radius.
  - 4. That all right angles are equal.
  - 5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.
- From the 5. it immediately derives that two parallel lines, if produced indefinitely, never meet.

The fifth postulate in the Greek tradition

- The fifth postulate however, excited also in the Hellenic age a strong debate because of its lack of **empirical evidence,** supported rationally also by the **rational evidence** that that many so-called «asymptotic» lines that do not meet in the finite space, meet instead in the infinite space (for example, a hyperbola is asymptotic at the axes).



The odd destiny of the negation of the fifth postulate in the *Corpus Aristotelicum*

- Given that the fifth postulate did not seem «self-evident» at all, many mathematicians, both ancient and modern, felt the need to demonstrate its truth, by deducing it from the other four postulates.
- Imre Toth, a Romanian historian of mathematics, recently found (Toth 1998) a lot of passages in the *Corpus Aristotelicum* in which Aristotle suggests the possibility of avoiding the fifth postulate.
- What is odd is that **all these passages,** starting from the fundamental one of Aristotle's *Eudemian Ethics* (1222b15 − 37, repeated in the post-Aristotelian epitome *Magna moralia* 1187a36 − b3), are **not in a logical or mathematical context**, but in a **deontic one,** as an example of moral choice of **a bad ethical choice,** where the alternative is between the Euclidean (good) and a non-Euclidean (bad) triangle (!). This is interpreted by Toth as a hint of a non-Euclidean axiomatic interpretation of geometry
  - As emphasized by a more attentive exegesis of the Edemian passage, the mistake of Toth interpretation is that Aristotle in these passages is not referring to the choice between two axioms - and hence between a Euclidean and non-Euclidean geometry axiomatics – but between two **different types of premise-consequence relationship,** deductive **(direct implication:** $p \rightarrow q$) inductive that uses the **converse implication:** $p \leftarrow q$) like the **deontic reasoning (*diaresis*)** where it is as a function of **the goal *q*** we want to pursue, and/or of **the theorem *q*** we want to demonstrate, **that we necessarily pose the principle *p* sufficient** for pursuing in ethics (deriving in logic) *q*. So if *q* changed (non-Euclidean triangle) also *p* would have to change (negation of the fifth prosulate) proportionally (and viceversa).
  - We come back to this point in the Basti2 complementary course because it is only in the context of a **modal logic analysis –** firstly developed by Aquinas in the Middle Age **–** of the **foundation of the formal nexus of necessity (logical entailment)** in different **semantics** (logical, ontological, deontic, epistemic) that this Aristotelian «oddity» as to the V postulate reveals its relevance, otherwise it would be trivial.

The prodromes of the crisis of foundations of modern mathematics

- Anyway, the development of **modern calculus** through the (initial) **algebraization of geometry** by Descartes, and the definition of modern **infinitesimal calculus** by the Newtonian **method of fluxions** as extension of Leibniz **calculus of the derivatives,** is the **ideal bridging** between the Archimedes apex of Greek mathematical physics and the modern Galileian science.

- However, the modern Kantian foundation of mathematical truths, and specifically of calculus, by his **transcendental interpretation** of Descartes' **evidence principle,** was deeply troubled by the almost contemporary discovery of the **non-Euclidean geometries** as intrinsic completion of the algebraic interpretation of geometry.

- Indeed, the supposed self-evidence of Euclidean **fifth postulate** was always problematic since its early Greek origins → ancient and modern failures of demonstrating it, till G. Saccheri (1667-1773) definitive result of the impossibility of demonstrating it even ***per absurdum*** → **effective possibility of existence** of non-Euclidean geometries, a program initially fulfilled by N. J. Lobachevski's (1793-1856) **hyperbolic geometry,** and by J. Bolyai's (1802-1860) **absolute geometry,** and completed by B. Riemann's (1826-1866) full algebraization of geometry, starting from his famous *Habiltationschrift* on the foundations of geometry (1854, publ. In 1868): «**On the Hypotheses which uderlie geometry».**

The hypothetical character of mathematics and the problem of foundations

- The logical and ontological consequences of the recognition of the **hypothetical, non-apodictic character** of mathematics in its pured and applied uses can be so summarized:
  - “The traditional belief that the axioms of geometry (or, for that matter, the axioms of any discipline) can be established by their apparent self-evidence was thus radically undermined. Moreover, it gradually became clear that proper business of pure mathematicians is to ***derive theorems from postulated assumptions***, and that it is not heir concern whether the axioms assumed are **actually *true”*** (Nagel & Newman 1993, 21).
  - “Lobachevski is considered the «Copernicus of geometry», the one who transformed this domain of mathematics by creating an entirely new branch (...), *by showing how Euclidean geometry was not the exact science, repository of absolute truth, that it had been previously considered.* In some sense we could say that the discovery of non-Euclidean geometry gave a mortal blow to Kantian philosophy, that was comparable to the consequences that the discovery of incommensurable magnitudes had had on Pythagorean thought (see *supra* § 1.1). Lobachevski’s work made it necessary *to radically modify the fundamental ideas on the nature of mathematics*” (Boyer 1968, 621ff, emphasis added).

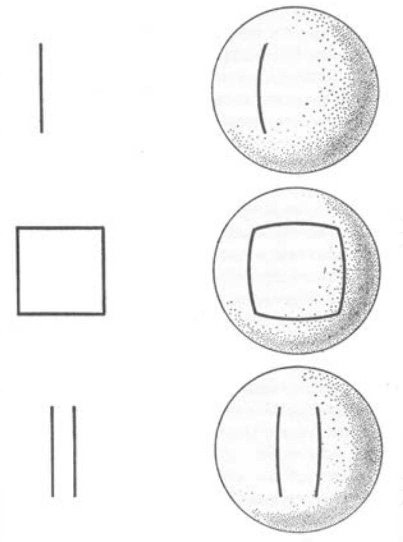
Riemann’s completion of the algebraization of geometry started with Descartes

- Descartes’ elementary algebraization of geometry allowed one to turn Euclidean postulates of plane geometry into as many **algebraic expressions**. The «point» corresponds with a pair of numbers; the «line» with a (linear) relation between numbers described by a first degree equation in two unknowns; the «square» with a relation between numbers described by a second degree equation of a specific form, etc.
- Riemann’s geometry introduced **a further level of abstraction from intuitive contents**. The algebraic expressions of elementary algebra still refer to magnitudes (numerical and/or spatial) that they denote in a symbolic form. After Riemann, the algebraic expressions used in his geometry are constructed in such a way that **they denote nothing**, they **are expressions devoid of any (referential) meaning**, but that can take on different (referential) meanings through a **specific *interpretation (model*)** by adding further axioms to the formal system.
- Stated otherwise, through its algebraization geometry becomes a deductive science, able to represent abstractly, with its symbolic, algebraic formalism, ***relations* and *structures***, rather than **relations among continuous quantities** as it was in the past. These relations and structures can be applied, through subsequent interpretations, to multiple types of objects.

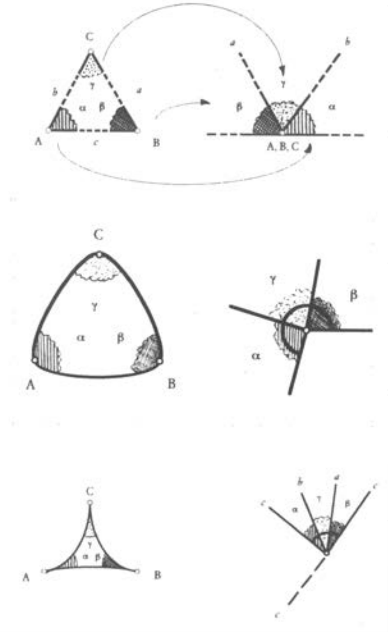
The problem of consistency of axiomatic systems

- **In apodictic systems**, indeed, because it is evident that **true axioms** must be **compatible** (i.e., non reciprocally contradictory), all the theorems coherently derived from them are compatible too, so that the **system is consistent** (i.e., it does not include contradictory statements).
- In **hypothetical systems** however, in which the **absolute truth** of axioms falls → the problem of **proofing the system consistency** becomes dominant, givent that it cannot be automatically granted by the soundness of the axioms.
- Riemann's **partial solution** of this problem consists in constructing a **model** of his ellyptical geometry on an **Euclidean sphere** (i.e., a convex space with **positive curvature**), extended by **Beltrami** to the **Euclidean pseudo-sphere** (i.e. a concave space with **negative curvature**) for modeling Lobacevskji's hyperbolic geometry:
  - → **Relative consistency** of the non-Euclidean geometries as to the Euclidean one;
  - → **Intuitive representation** of some otherwise counter-intuitive statements of the non-Euclidean geometries → **intuitive proof** of their consistency by such **a finite modeling** (see next slide)**.**
- → The double problem remains of: 1) constructing **absolute proofs** of consistency for axiomatic systems; 2) the impossibility of the use of **intuitive proofs** for axioms requiring **infinite models** such as the axioms of **mathematical analysis** and of **number theory.**

Intutive proofs of some non-Euclidean geometries statements by their finite modeling



Two points on flat space (left side) become two points on Euclid’s sphere (right side); two pairs of parallel straight lines become two maximal circles; two parallel segments become two arcs of *maximal circle**. If prolonged, these meet: clearly, this is contrary to what is stated in Euclid’s axiom of parallel lines.



Intuitive description of the triangles, respectively: in Euclidean flat plane (top), in Riemann’s elliptic plane (centre) modelled on Euclid’s sphere, in Lobachevski’s hyperbolic space (bottom) modelled on Beltrami’s pseudosphere. Beside each figure, the projections of the three angles obtained by superimposing the adjacent sides of the original triangles show that, in the first case, the sum of the internal angles equals two right angles (180°); in the second case, it is greater than 180°; in the third, it is lesser.