Uniwersytet
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Towards a Contemporary Ontology

The New Dual Paradigm in Natural Sciences: Part I

Module 4: The QFT interpretation of quantum physics as “second quantization

Course WI-FI-BASTI1 2014/15

Introduction

Module 4: “The QFT interpretation of quantum physics as “second quantization”

Course modules

Modules	Topic	Suggested Readings
	SECTION ONE	
0.	*Introduction and Course Overview*	
1.	The Birth of Modern Science	Refs.: **1**, chs. 0, 1, 2.
2.	The Question of Truth in Modern Science	Refs.: **1,** chs. 3, 4.
	SECTION TWO	
3.	The Information Theoretic Interpretation of QM	Refs.: **4-10**.
4.	QFT interpretation as ‘Second Quantization’ and the Physics of the Condensed Matter	Refs.: **11**, pp. vi-xii, 1-35, 137-178; **12; 13**.
	SECTION THREE	
5.	The (Co)Algebraic Interpretation of QFT as q-deformed Hopf Algebra / Coalgebra	Refs.: **11,** pp. 131-185; **14**.
6.	The DDF Principle of QFT, its Cosmological Relevance and Its Ontological Interpretation	Refs.: **14-19**; **1**, ch. 5.
	SECTION FOUR	
7.	Universal Coalgebra and the Interpretation of QFT Systems as STS	Refs.: **16**; **20**
8.	*Conclusions*	

Bibliography

Bibliography of the Module 4

Bibliography

- Main References:
  - G. Basti, *Philosophy of Nature and of Science, vol. 1: The foundations,* transl. by Philip Larrey, Rome 2012 (for student use only), ch. 2 [attached]
  - G. Basti, *QFT: An Evolutionary Interpretation Of Nature From Cosmology To Neuroscience* [Lecture Notes:attached].
  - M. Kuhlmann, «Quantum field theory». In: *The Stanford Encyclopedia of Philosophy* (Spring 2014 Edition), Edward N. Zalta (ed.), URL = http://plato.stanford.edu/archives/spr2014/entries/quantum-field-theory/ (accessed 29/10/2014).
- Other References:

1. J. EARMAN, D. FRASER, "Haag's Theorem and Its Implications for the Foundations of Quantum Field Theory", *Erkenntnis* 64(2006), 305-344 [attached]
2. M. BLASONE, P. JIZBA, G. VITIELLO, «Preface», in: *Quantum field theory and its macroscopic manifestations. Boson condensations, ordered patterns and topological defects* , Imperial College Press , London, 2011, pp. vii-xii.
3. G. VITIELLO, «Links. Relating different physical systems through the common QFT algebraic structure», *Lecture Notes in Physics*, 718 (2007), 165-205 [attached].

Bibliography II

3. J. Rutten, “Universal coalgebra: a theory of systems”, *Theoretical computer science,* 249,1(2000), pp. 3-80 [attached].

4. B. Jacobs & J. Rutten, “An introduction to (co)algebra and (co)induction” in: *Advanced topics in bisimulation and coinduction,* D. Sangiorgi & J. Rutten (Eds.), Cambridge UP, Cambridge UK, 2012, pp. 38-99.

5. P. W. Anderson, “More is different”, *Science*, New Series, 177,4047 (1972), pp. 393-396 [attached].

Module 4

QFT interpretation as ‘Second Quantization’ and the Physics of the Condensed Matter

From the standard QFT to algebraic QFT to dissipative QFT. I

- The QFT is actually **a family of theories** and not only one.

1. The **standard interpretation of QFT (OQFT)** or «second quantization» is an extension of QM to **many body physics** developed originally by Dirac, Fock and Jordan.
   - It is based on the **indistinguishability of particles** in QM, differently from CM where each particle is defined by its own position vector $\mathbf{r}_i$ → **different $\mathbf{r}_i$'s configurations** → different many body states.
   - In QM **(first quantization)** exchanging two particles does not change the quantum state, $\mathbf{r}_i \leftrightarrow \mathbf{r}_j$, → **the same wave function Ψ is invariant** for particle exchange, **symmetric** in the case of bosons (photons, gluons, etc.), **anti-symmetric** in the case of fermions (quarks, electrons, etc.):

$$\psi_B(\ldots,\mathbf{r}_i,\ldots,\mathbf{r}_j,\ldots) = +\psi_B(\ldots,\mathbf{r}_j,\ldots,\mathbf{r}_i,\ldots)$$
$$\psi_F(\ldots,\mathbf{r}_i,\ldots,\mathbf{r}_j,\ldots) = -\psi_F(\ldots,\mathbf{r}_j,\ldots,\mathbf{r}_i,\ldots)$$

From the standard QFT to algebraic QFT to dissipative QFT. II

- Before all a short explanation of bra-ket notation in QM and QFT as a stndard notation for describing quantum states. In QM the notation $\langle\varphi|\psi\rangle$ **consists of two parts:** the left part, $\langle\varphi|$**, the «bra»**, the right part, $|\psi\rangle$**, the «ket».** So the overlap expression $\langle\varphi|\psi\rangle$ typically means in QM the probability amplitude for the state ψ **to collapse into** the state φ**.**
- The **OQFT** overcomes the difficulties intrinsic to QM for dealing with many-body systems (redundancies in defining in which state the single particle is), because the problem becomes which is the number of particles occupying the same quantum state → many-body state represented in terms of **occupation number of particles** in the single quantum state (or **Fock state**), i.e.,

$$|[n_\alpha]\rangle \equiv |n_1, n_2, \ldots, n_\alpha, \rangle$$

- With

$$n_\alpha = \begin{cases} 0,1 & \text{for fermions} \\ 0,1,2,3\ldots & \text{for bosons} \end{cases}$$

- The Fock state with all occupation numbers being zero is the **vacuum state** $|0\rangle$. By applying many times the **creation/annihilation operators** to the vacuum state, we can add/delete as many particles to the vacuum state.
- All the Fock states $|[n_\alpha]\rangle$ form the basis of the **many-body Hilbert space** or **Fock space** → any generic quantum many-body state is expressed as a **linear combination of Fock states.**

From the standard QFT to algebraic QFT to dissipative QFT. III

- **Problem:** for maintaining the coherence of the wave function it is necessary to eliminate all interactions with environment just as in CM, and hence to conceive the many-body system as **isolated** → the quantum vacuum (QV) ground state at 0°K (absolute zero).
- This ilimit of standard QFT is formally confirmed by **Haag's theorem** demonstrating that Dirac's «interaction representation (picture)» does nor hold in QFT, so that also Stone-Von Neumann theorem does not hold → **infinitely many inequevalent CCR's (= infinitely many degrees of freedom) in QFT.**

1. This is against evidence that **QV temperature at the ground state is >0,** an evidence emerging from cosmology ('tHooft-Susskind holographic universe model) and strictly connected with Third Principle of Thermodynamics
2. Recent evidence (March 2014) of the existence of **gravitational waves** on the cosmic micro-wave background radiation (CMBR) of the universe depending on the **QV energy.**

- → Necessity of developing a **dissipative QFT** interpreting dynamically **the infinitely many degrees of freedom** that in any FT necessarily appear, as a very natural description of a «hot» QV ground state with all the energy «bounded» (no free energy), just as in classical thermodynamics → SSB's of QV as inducing **phase coherences** and not **state coherences,** i.e., **structures defined on oscillating fields and not on static points**.
- An **intermediate step** toward this direction was the development of an **algebraic interpretation of QFT** .

From the standard QFT to algebraic QFT to dissipative QFT. IV

2. → **Several algebraic interpretations of QFT** all based on the Stone-Von Neumann theorem (1931) (Kuhlmann, 2014):
   - In the context of QM, Schrödinger, Dirac, Jordan and von Neumann realized that Heisenberg's matrix mechanics and Schrödinger's wave mechanics are just **two (unitarily) equivalent representations of the same underlying abstract structure**, i.e., **an abstract Hilbert space $\mathcal{H}$ with linear operators** acting on it .
   - I.e. they are **two different ways for representing the same physical structure**, and it is possible to switch between these different representations by means of a unitary transformation, i.e. an operation that is analogous to an innocuous rotation of the frame of reference.
   - **Representations** of some given algebra or group are **sets of mathematical objects**, like numbers, rotations or more abstract transformations (e.g. differential operators) **together with a binary operation** (e.g. addition or multiplication) that combines any two elements of the algebra or group, such that the structure of the algebra or group to be represented **is preserved**.

From the standard QFT, to algebraic QFT, to dissipative QFT. V

- **The Stone-Von Neumann Theorem:** In 1931 von Neumann gave a detailed proof (of a conjecture by Stone) that **the canonical commutation relations (CCRs)** for position coordinates and their conjugate momentum coordinates in the configuration space, fix the representation of these **two sets of operators in $\mathcal{H}$ up to unitary equivalence** (von Neumann's uniqueness theorem) → **finitely many unitary equivalent representations of CCRs.**
- This means that the specification of **the purely algebraic CCRs suffices to describe a particular physical system → Algebraic QFT (AQFT):** what matters in QFT are not the fields or the particles, but the underlying algebraic structures (for a synthesis, see (Kuhlmann 2014, §4.2) .
- **Problem:** in QFT the Stone-Von Neumann theorem **does not hold!** Because of the presence of infinitely many degrees of freedom → **infinitely many unitary inequivalent representations (UIR's) of CCRs** → "sticking the usual Hilbert space formulation tacitly implies **choosing** one particular representation", one particular algebra (Kuhlmann 2014, §4.2). → A **physical** QFT must give a **dynamic (causal) justification of this choice,** not for giving up the richness of the algebraic formalism, but for reinforcing it with a **coalgebraic** «partnership».

- → **UIR's are the core problem** of any standard or algebraic QFT (Kuhlmann 2014), at least till we do not consider the alternative view of a **dissipative QFT** (Vitiello 2007).

From the standard QFT, to algebraic QFT, to dissipative QFT. VI

3. The further step toward a dissipative QFT is the notion of **quantum groups** introduced by the Field Medal Ukrainian mathematician **V. Drinfeld,** and independently by the Japanese mathematician **M. Jimbo** in late 60's of XX cent., as a mathematical tool for connoting **non-commutative algebras,** and a particular class of **Hopf algebras**: the **q-deformed Hopf algebras,** where «q» is for «quantum».

- Hopf algebra is a classical tool used in CM and in QM (e.g., for representing two equivalent quantum states), since it is a **bialgebra.** I.e., a Hopf algebra is **isomorphic** with its coalgebra and hence perfectly suitable for representing algebraically the **commutative character** of all compositions characterizing all physical **symmetries** and **laws**.
- → The q-deformation parameter is defined also as a **squeezing parameter** because **twisting** the perfect circularity of the Hopf bialgebra symmetrical structure, making **non-commutative its compositions, anyway preserving its structure** → extension of **symmetry** notion also to quantum non-commutative relations.
- What is important to emphasize is that *q*-algebras and *q*-groups are **one of the two admitted deformations** allowed by the **Weyl-Heisenberg algebra (WH)** playing the role of **superalgebra** in quantum physics, so to justify **supersymmetry theories.**
- The other deformation admitted by WH is the so-called ***q*-WH** extending the **structure-preserving** non-commutative character of the relations also to **coproducts** , so to extend Drinfeld notion of *q*-deformed Hopf algebras to the duality ***q*-deformed Hopf algebras/coalgebras** where **a structure preserving mapping** (homomorphism) is defined for inversion of all the compositions order **and** of all the **arrows** defining the (co-)products. For this reason ***q*-WH is connoted also as a Hopf-superalgebra.**.
- In this way, the isomorphism of a Hopf bialgebra becomes a **homomorphism** (i.e. bijective without being isomorphic) between a q-deformed Hopf algebra ($\mathbb{H} : H \times H \to H$) i.e. where all morphisms «→» are **surjective,** and its dual q-deformed Hopf coalgebra: ($\mathbb{H}^{op} : H \to H \times H$), i.e. where all morphisms are **injective**. Anyway, because in any bijective relation the **structure is preserved, the truth is preserved too = «duality» algebra-coalgebra in Category Theory** (any true assert defined on an algebra is true also on its dual coalgebra, and vice versa).

The proper of the dissipative QFT

- → In the dissipative QFT formalism the "symmetry squeezing" parameter *q* of the *q*-deformed Hopf algebra represents **the energy injected from the thermal bath** with which the system is **entangled**, while the thermal bath of the system is represented by the dual *q*-deformed Hopf coalgebra from which **the energy is subtracted**.
  - Hence the **arrow inversion** characterizing their homomorphism is **the energy arrow,** and then the **far from equilibrium stability** of such a "doubled **entangled** coherent state", effectively the **phase coherence domain of the oscillating fields** charactering the whole system, and corresponding to a **SSB of the QV**
- However, because in any case – even though far from thermodynamic equilibrium – the energy balance must be conserved, what effectively is elicited by a SSB is **a doubled-mode *N-N** interaction**, where the latter *N** is **the mirroring, twisted image** – in the coalgebraic sense – of the structure of the former characterized by *N*, where |*N*| is the value of the condensate of the **Nambu-Goldstone bosons** characterizing each SSB and hence each derived phase coherence.
- This corresponds to **a local collapse of the number of the degrees of freedom (≃UIR's number)** in the QV substrate, matched by the **doubling of the reduced degrees of freedoms** in its mirroring image → **dynamic choice of an algebra,** i.e., of a subset of the infinitely many UIR's of the CCR's characterizing the QV → the Hamiltonian character of a quantum system, and hence Hilbert's space formalism, is recovered by considering each quantum system as **entangled with its thermal bath.**

QFT main ontological issues

- Kuhlmann emphasizes three main **ontological interpretations** about the «ontological primacy» in QFT (Kuhlmann 2014, §5) with their problematic issues:

1. **Particles *vs.* fields** ontological primacy, typical of all OQFT's:
   - **Against particle primacy**: double issues of **localizability** in the QFT relativistic realm, and **countability** because observer dependent (Unruh effect). Moreover, in QFT their representations are **unitarily inequivalent** as to a Fock space (Haag's theorem);
   - **Against field primacy:** this interpretation suffers the same limitations because «quantum states» are **operator-dependent** → it is impossible **to ascribe properties to points in space that are typical of the «field» notion**. Moreover the «wave functional» suffers the same problem of UIR as to a Fock state, also if the occupancy numbers are defined in terms of **field quanta** instead of **particles**.
2. **The structure** primacy as to particles or fields, typical of all AQFT's:
   - Apart from the **Platonistic flavor** of such an ontology, **the problem remains** because effectively a structure has to be always ascribed either to particle sets or to fields.
3. **The disposition tropes (=changeable properties)** ontological primacy, proposed by Kuhlman himself:
   - **Given that the infinite UIR** physical and mathematical interpretation is **the unresolved issue** of all OQFT's and AQFT's interpretations → the idea is to give primacy by **superselection** to a subset of the infinitely many UIR's that are **irreducible,** and hence **essential** for characterizing the **algebra of observables** of a quantum state either particle-like or field-like.
   - → **Modal character** of such an ontology: distinction between **essential – non-essential** properties/tropes; **dispositional character** of tropes. **However,** the problem remains of the justification of the **superselection among UIR's .**

The solution of QFT ontological issues

- Given:

1. The interpretation of the infinitely many UIR's in QFT in terms of **QV infinitely degrees of freedom** in the infinite volume limit**;**
2. The strict relationship between the infinitely many UIR's and the infinitely many **SSB's of a «hot» QV ground state** not-representable in QM.
3. The strict relationship between a SSB and a ***q*-deformed Hopf algebra/coalgebra doubling,** applied to fields (doubled, entangled phase coherence domains) and not to points in space (coherent spaces) like in AQFT;
4. The necessity of interpreting the *q*-deformation parameter as the **selector,** as far as a **thermal parameter** derived by the convergent necessity of **1)** supposing **a finite temperature of the vacuum** ground state **different from 0,** both in cosmological applications of QFT**,** and in condensed matter applications of QFT; → **2)** inadequacy of the *q*-WH interpretation of the **squeezing parameter *q*** for reducing the **density matrix** of the coherent state (i.e., the so-called **decoherence to a point-like state (=Dirac delta)**) because we are faced here not with state-transitions but with **phase-transitions,** i.e., transitions between **phase-coherence domains (matrix reduction by squeezing corresponds to the calculus of the matrix determinant (= Kroenecker delta));**

- → **QFT interpretation** of particle-field duality in **dynamical (causal) terms,** related with a SSB of the QV at the ground state and each correspondent with the emergence of a **phase coherence domain** characterizing the **collective, no-longer individual, behavior («more is different!»)** of the oscillating particles (quanta of the relative fields) involved**:**

QFT interpretation of particle-field duality

- This means rewriting in QFT, the QM uncertainty principle:

$$\Delta x \Delta p \geq \hbar/2$$

- For large amplitude coherent domains of phase θ, as the following:

$$\Delta n \; \Delta\varphi \geq f(\theta)$$

Where $f(\theta)$ is different from zero, n is the number of quanta of the force field, and φ is the field phase. If ($\Delta n = 0$), φ is undefined so that it makes sense to neglect the waveform aspect in favour of the individual, particle-like behaviour. On the contrary if ($\Delta\varphi = 0$), n is undefined because an extremely high number of quanta are oscillating together according to a well-defined phase, i.e., within a given phase coherence domain. In this way, it would be nonsensical to describe the phenomenon in terms of individual particle behaviour, since the collective modes of the force field prevail.

- **Compatibility** of such a physics with Aquinas' ontology of a **composed substance** in which the elements – that were formerly as many actually existing individual substances – lose their individuality, as far as become **parts virtually existing** in the new **actually existing whole,** whose properties are irreducible to those of the components (see the famous paper of the Nobel Laureate P.W. Anderson **"More is Different",** relating the emergence of unreducible totalities, to the SSB of the QV (Anderson 1972) **.**