Uniwersytet
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Towards a Contemporary Ontology

The New Dual Paradigm in Natural Sciences: Part I

Module 5: The (Co)Algebraic Interpretation of QFT

Course WI-FI-BASTI1

2014/15

Introduction

Module 5: “The (Co)Algebraic Interpretation of QFT as q-deformed Hopf Algebra / Coalgebra””

Course modules

Modules	Topic	Suggested Readings
	SECTION ONE	
0.	*Introduction and Course Overview*	
1.	The Birth of Modern Science	Refs.: **1**, chs. 0, 1, 2.
2.	The Question of Truth in Modern Science	Refs.: **1,** chs. 3, 4.
	SECTION TWO	
3.	The Information Theoretic Interpretation of QM	Refs.: **4-10**.
4.	QFT interpretation as 'Second Quantization' and the Physics of the Condensed Matter	Refs.: **11**, pp. vi-xii, 1-35, 137-178; **12; 13**.
	SECTION THREE	
5.	The (Co)Algebraic Interpretation of QFT as q-deformed Hopf Algebra / Coalgebra	Refs.: **11,** pp. 131-185; **14**.
6.	The DDF Principle of QFT, its Cosmological Relevance and Its Ontological Interpretation	Refs.: **14-19**; **1**, ch. 5.
	SECTION FOUR	
7.	Universal Coalgebra and the Interpretation of QFT Systems as STS	Refs.: **16**; **20**
8.	*Conclusions*	

Bibliography

Bibliography of the Module 5

Bibliography

- **Main References**:
  - G. BASTI, *Philosophy of Nature and of Science, vol. 1: The foundations,* transl. by Philip Larrey, Rome 2012 (for student use only), ch. 2 [attached]
  - G. BASTI, *QFT: An Evolutionary Interpretation Of Nature From Cosmology To Neuroscience* [Lecture Notes:attached].
- **Other References**:

1. M. BLASONE, P. JIZBA, G. VITIELLO, «Preface», in: *Quantum field theory and its macroscopic manifestations. Boson condensations, ordered patterns and topological defects* , Imperial College Press , London, 2011, pp. vii-xii.
2. G. VITIELLO, «Links. Relating different physical systems through the common QFT algebraic structure», *Lecture Notes in Physics*, 718 (2007), 165-205 [attached].

Bibliography II

3. Y. VENEMA, “Algebras and coalgebras”, *Handbook of modal logic, Studies in Logic and Practical Reasoning, volume 3*, Patrick Blackburn (Editor), Johan F.A.K. van Benthem (Editor), Frank Wolter (Editor), Elsevier, Amsterdam, pp.331-426 (cap. VI), 2007 [attached]

4. J. RUTTEN, “Universal coalgebra: a theory of systems”, *Theoretical computer science,* 249,1(2000), pp. 3-80 [attached].

5. D. SANGIORGI, «On the origins of bisimulation and coinduction», in: *ACM Transactions on Programming Languages and Systems,* **31**(4), 2009, pp. 111-151 [attached].

6. B. JACOBS & J. RUTTEN, “An introduction to (co)algebra and (co)induction” in: *Advanced topics in bisimulation and coinduction,* D. SANGIORGI & J. RUTTEN (EDS.), Cambridge UP, Cambridge UK, 2012, pp. 38-99.

7. G. BASTI, *Some principles for a QFT optical computing* (draft) [attached].

Module 5

The (Co)Algebraic Interpretation of QFT as q-deformed Hopf Algebra / Coalgebra

Algebra/Coalgebra Duality in Category Theory I

- Recently, in the last thirty years, but overall since 2000, year of publication of the fundamental work of J. Rutten, *Universal Coalgebra. A theory of systems* (attached), a convergent research on the theory and applications of **coalgebra** (effectively, **functor-coalgebra, or F-Coalgebra**) have been developed from three apparently very distant fields.
- The three fields are: 1) **Theoretical Computer Science** (TCS); 2) **Theoretical Physics** (QFT); 3) **Formal Philosophy** (FP).
- To understand the notion of **doubling of the degree of freedom (DDF) in QFT** let us start from the notion of "duality" in **category theory** (CT).
- In category theory, an abstract branch of mathematics, an **equivalence of categories** is a relation between two categories that establishes that these categories are "essentially the same". There are numerous examples of categorical equivalences from many areas of mathematics.

Algebra/Coalgebra Duality in Category Theory II

- Establishing an equivalence involves demonstrating strong similarities between the mathematical structures concerned. In some cases, these structures may appear to be unrelated at a superficial or intuitive level, making the notion fairly powerful: it creates the opportunity to "translate" theorems between different kinds of mathematical structures, knowing that the essential meaning of those theorems is preserved under the translation.

- An equivalence of categories consists of a **functor** (mapping conserving the structure) between the involved categories, which is required to have an **"inverse" functor.**

- However, in contrast to the situation common for **isomorphisms** (biunivocal correspondence) in an algebraic setting, the composition of the functor and its "inverse" is not necessarily the identity mapping. **Thus one may describe the functors as being "inverse up to isomorphism".**

Algebra/Coalgebra Duality in Category Theory III

- If a category is equivalent to the **opposite (or dual)** of another category then one speaks of a **duality of categories**, and says that the two categories are **dually equivalent**.
- **Duality** is a correspondence between properties of a category C and so-called **dual properties** of the **opposite category** C^{op}. Given a statement regarding the category C, by interchanging the **source** and **target** of each morphism as well as interchanging the order of **composing** two morphisms, a corresponding dual statement is obtained regarding the opposite category C^{op}.
- **Doing the reversal twice yields the original category**, so the opposite of an opposite category is the original category itself. In symbols, $(C^{op})^{op} = C$. When a category is equivalent with its own opposite it is **self-dual.** This is the case of the Hopf algebras, without deformations.
- **Duality**, as such, is the assertion **that truth is invariant under this operation on statements**. In other words, if a statement is true about C, then its dual statement is true about C^{op}. Also, if a statement is false about C, then its dual has to be false about C^{op}

Algebra/Coalgebra Duality in Category Theory IV

- **For instance:**
  - A morphism $f : A \rightarrow B$ is a **monomorphism** if $(f \circ g = f \circ h) \rightarrow (g = h)$ [left-cancelling morphism]. Performing the dual operation, we get the statement $(g \circ f = h \circ f) \rightarrow (g = h)$ For a morphism $f : B \rightarrow A$, this is precisely what it means for f to be an **epimorphism**. In short, the property of being a **monomorphism** is dual to the property of being an **epimorphism**.
  - Applying duality, this means that a morphism in some category C is a monomorphism **if and only if** the reverse morphism in the opposite category C^{op} is an epimorphism.
  - An example comes from reversing the direction of inequalities in a **partial order**. So if X is a set and $\leq$ a partial order relation, we can define a new partial order relation $\leq_{new}$ by $x \leq_{new} y$ if and only if $y \leq x$.
  - E.g., In C the partial ordering **ancestors/descendants** among some elements of a set X holds **iff** in C^{op} the opposite partial ordering **descendants/ancestors** holds too.
  - Also from this simple notions, it is evident that **the algebraic formalism of QFT**, in the dissipative case, **requires for being true the duality algebra/coalgebra.**

Algebra/Coalgebra Duality in Category Theory IV

- Finally it is evident that **the elementary language of category theory** is a two-sorted **first order language,** with **objects** and **morphisms** as distinct sorts, together with the relations of an object being the source or target of a morphism, and a symbol for composing two morphisms.
- For this reason we have to distinguish the **coalgebra** notion in **abstract mathematics, from the F-coalgebras** we are here referring to.
- **In abstract mathematics,** coalgebras are **dual** to **unital associative algebras** (e.g., unital rings) in the sense that the **axioms** of unital associative algebras can be formulated in terms of **commutative diagrams**. Turning all arrows around, one obtains the axioms of coalgebras. Of course, for non-unital algebras the dual is not a coalgebra. Neither a first order language in CT is able to deal with these **axiom reversals** characterizing such objects
- **In TCS and in theory of the dynamical systems (TDS)** we use **functor coalgebras (F-coalgebras),** suitable for **formally justifying**, in **TDS** a thermal field interpretation of QFT, **able to satisfy Haag's theorem**; in **TCS** a coalgebraic **semantics** of Boolean algebras (equation logic), also in terms of automata's reactive behavior (state transition systems, **STS**). Finally, in **FP** a complete modal relational semantics, able to justify **the ontological bi-conditional**. A fundamental step for restituting metaphysics **a scientific dignity in a post-modern age**!
- In all these cases the categories **F-algebras/F-coalgebras,** with their **endofunctors** Ω^*/Ω are **dually equivalent, i.e., Alg(Ω^*)** $\leftrightarrows$ **Coalg(Ω)** (Venema 2007, 416)**.**

Bogoliubov transform and the dissipative QFT

- We offer in Basti2 a **coalgebraic interpretation of modal logic,** where, as far as defined on Aczel's **non-well founded sets,** it is able to justify the existence of **homomorphisms between coalgebras** in terms of **bisimilarity and/or of observational equivalence,** and a **coalgebraic theory of proof** based on the powerful notion of **coinduction** as dual to algebraic induction.
- As to the present aim of justifying a **dissipative QFT** as **the proper QFT** because satisfying Haag's theorem, we have to introduce the key notion of Bogoliubov transform (BT), that offers an essential contribution to the study of **the commutation/anticommutation relations for bosonic/fermionic creation and annnihilation operators** in a QV with a **temperature >0.**
- Such a transform, since it was introduced by the Russian mathematician **Nikolay Bogoliubov** (1929-1992) in 60's**, is a fundamental ingredient of QFT** in condensed matter physics, with applications in **superconductivity** and **ferromagnetism,** that is for studying **system able to pass through several stable phases,** as far as these passages are strictly related with the **thermal parameter, θ.**
- More recently such a transform had an essential role in demonstrating the **existence of Hawking radiation for the black-holes,** and hence with the emergence of the **Berkenstein-Hawking entropy** we already introduced, as an essential ingredient of present cosmology**.** What is more important for us is that, after the action of Bogoliubov operators of creation/annihilation, **the QV ground state |0⟩** is modified by the determination in it of **squeezed coherent states** correspondent to as many **phase coherence phenomena in it.**

q-deformed Hopf algebras in a thermal QV condition

- As Vitiello emphasizes, all this explains why the squeezed coherent states intrinsically related to the **q-deformed Hopf algebras** were irrelevant, when introduced in the context of the **closed quantum systems** of QM, or of the algebraic interpretation of QFT studied in the precedent module.
- Without a thermal vacuum, the Stone-Von Neumann theorem holds, and the infinitely many inequivalent representations of the CCR cannot occur, despite they are previewd by the Haag theorem, are related with the **spontaneous symmetry breakdowns (SSB)** of the QV, and were firstly revealed and defined rigorously by the BT creation/annihilation operators,.
- All this means that the **q-parameter** of the Hopf algebra deformation must be identified with the **thermal parameter θ of the BT,** so to allow the interpretation of the **duality q-deformed Hopf algebras/coalgebras,** with the intrinsic **reversal of the arrows and of the composition** just introduced, in terms of a **thermodynamic energy balance,** giving coalgebras formalism an essential role, not only in TDS, but in **fundamental physics.**